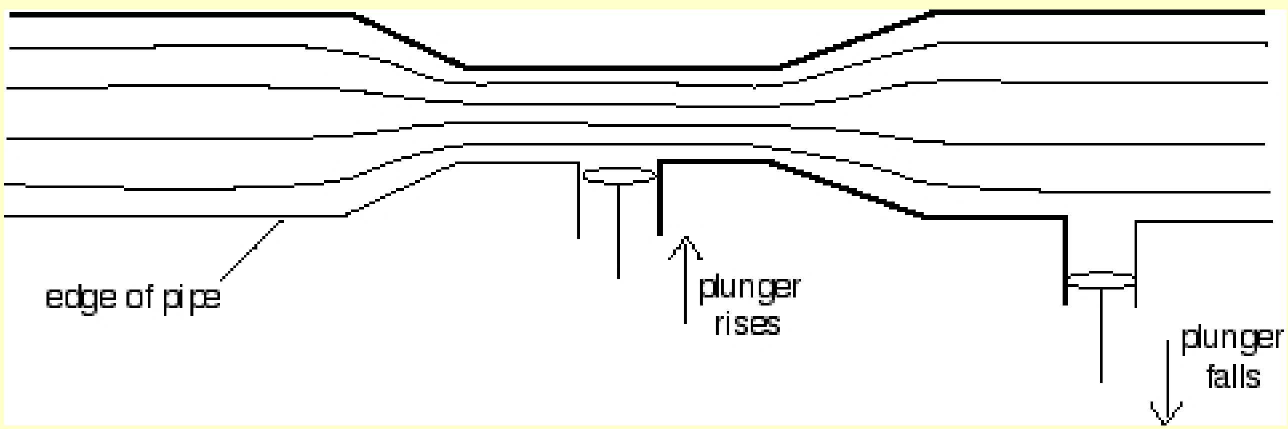
G432 – Architecture of Sedimentary Deposits

September 10, 2007

Grain movement and formation of sedimentary structures

The role of Bernoulli lift forces:

- Grain movement can be initiated or at least influenced by lift forces associated with the Bernoulli principal.
- Recall that Bernoulli's principal refers to the inverse relationship between velocity of fluid and pressure of fluid.



- **Derivation of the Bernoulli equation:**
  - Sum of energy in system is constant (First Law of Thermodynamics)
  - Kinetic energy + Pressure + Potential Energy = Constant

$$\frac{1}{2}\rho U^2 + P + \rho g y = \text{constant}$$

where ρ = fluid density and P = fluid pressure

We want to compare one point in the flow with another point in the flow:

$$\frac{1}{2}\rho_1 U^2{}_1 + P_1 + \rho_1 g_1 y_1 = \frac{1}{2}\rho_2 U^2{}_2 + P_2 + \rho_2 g_2 y_2$$

$$P_2 - P_1 = \frac{1}{2}\rho_1 U^2{}_1 + \rho_1 g_1 y_1 - \left(\frac{1}{2}\rho_2 U^2{}_2 + \rho_2 g_2 y_2\right)$$

Density of the fluid doesn't change, so

$$P_2 - P_1 = -\frac{\rho}{2}\left(U^2{}_1 + 2g_1 y_1 - U^2{}_2 - 2g_2 y_2\right)$$

$$\text{(or)}\ P_2 - P_1 = -\frac{\rho}{2}\left(U^2{}_2 + 2g_2 y_2 - U^2{}_1 - 2g_1 y_1\right)$$

g does not change, assuming constant altitude. Also, if we compare pressures at some constant height y, than g and y cancel.

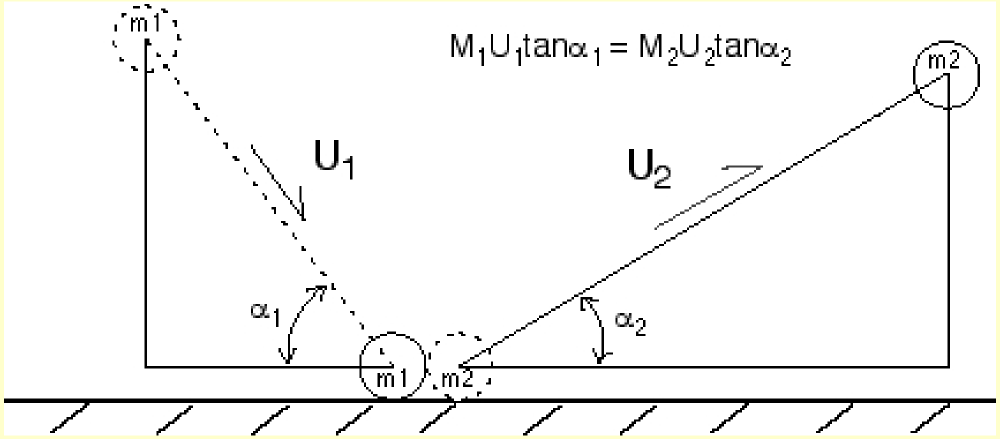
Thus the remaining equation becomes:

$$P_2 - P_1 = -\frac{\rho}{2}\left(U^2{}_2 - U^2{}_1\right)$$

The role of grain impacts on the bed:

Wind tunnel experiments have shown that grain movement can be started by artificially 'seeding' the bed with falling sand grains, provided that the flow velocities are high enough to approach the critical threshold needed for grain movement.

- As long as grains are artificially added to the bed, grain movement continues downflow. When 'seeding' stops, grain movement ceases.
- Thus, the 'impact threshold' of grain movement is different than the 'fluid threshold' of grain movement.
- Considering the Total Applied Shear Stress exerted on a bed from a moving fluid, it is important to recognize that impacting grains transmit a shear stress onto the bed, just as the moving fluid does.
- Consider a saltating grain of mass m impacting on a bed with velocity U and making an angle a with the bed surface. If elastic rebound occurs and neglecting frictional forces, the momentum of the incoming grain is transferred to the moved grain.
- Transfer of momentum normal to the bed: 2mUsina = P
- Transfer of momentum parallel to bed: 2mUcosa = T



- Bagnold (1954) was able to measure the magnitude of T and P in a set of experiments involving a rotating drum filled with a fluid and particles entrained in the fluid.
- He showed experimentally that when densely arrayed cohesionless grains are sheared together in a fluid, the time-averaged force generated by repeated grain-grain contacts or close encounters can be resolved into a tangential and a normal component.

  – Tangential component = T
  – Normal component = P
  – T/P = tanα = dynamic friction coefficient

- Bagnold's experiment is similar to taking a large hollow cylinder and putting some ping-pong balls in it.

  – **Experiment a)** Swirl the cylinder around long enough, the ping-pong balls move around the interior of the cylinder and stay in contact with the wall. Centrifugal force presses them to the side (this is the tangential force T), but there is apparently no strong force which requires them to move towards the center.
  – **Experiment b)** Ping-pong balls are glued to the interior of the cylinder to form a fixed, rough layer. Some loose balls are then added and the cylinder is swirled. Free balls now repeatedly bounce towards the center of the cylinder. Thus, there now appears to be a strong, inward-directed stress, corresponding to Bagnold's P.

- Thus, during bedload transport, we can infer that the total applied shear stress to the bed must have a fluid and a solid component.

G		Ts		T
Total applied shear stress	=	Fluid shear resistance	+	Solid shear resistance

- Bagnold's work showed that there is a great increase in the shear resistance of the solid/fluid mixture when compared to fluid alone.

- He defined two main regions of behavior, which he defined in terms of a dimensionless number, now called the **Bagnold number**.

  - 1) A viscous region of behavior at low strain rates and/ or low grain concentrations showed the ratio T:P to be constant at around 0.75.

    - Bagnold hypothesized that grain/grain effects were caused mainly by near approaches where a 'repulsion' of the grains went into force before the grains could actually collide. (Thus, the 'viscous behavior'.)

  - 2) An inertial regions at high strain rates and/or high grain concentrations where the ratio T:P tended to be constant at around 0.32. Grain/grain collisions dominated.

    - In aeolian systems, where ρquartz:rair = 2000:1, so much momentum of the fluid is tied up in grain saltation, that Ts (fluid shear resistance) is effectively 0, and G = T.

- Bagnold called P the **dispersive stress**, and he postulated that it should be in equilibrium with the normal stress due to the gravity weight force of the moving bedload.

Grain settling:

- As soon as grains are lifted off the bed surface, they begin to fall back to the bed.
  - A settling particle initially accelerates as it begins to settle through a fluid.
  - Eventually, the terminal velocity is reached when the drag force on the grain is balanced by the gravitational acceleration of the grain minus the buoyancy of the grain due to the fluid. For small grain sizes (>0.1mm), the terminal velocity of the settling grain determined through Stoke's Law.

$$U_g = \frac{2r^2 g}{9\mu}\left(\rho_s - \rho_f\right)$$

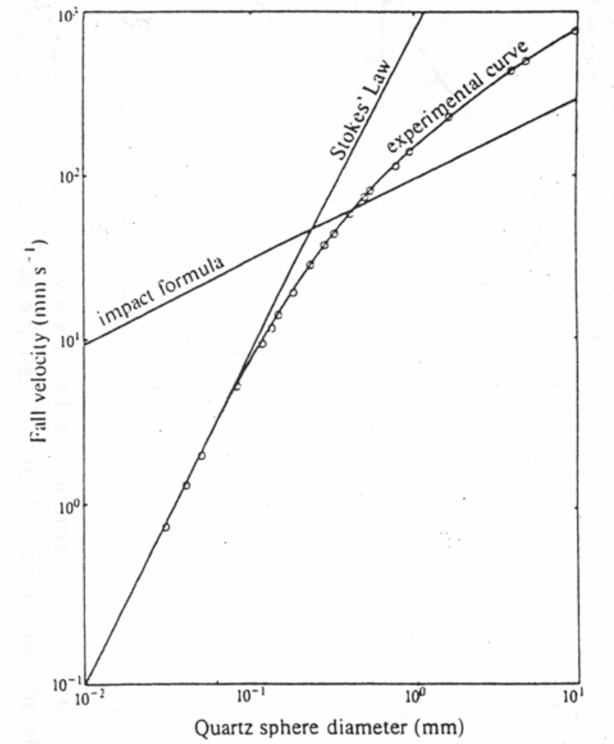


Figure 6.1 Graph to show fall velocity as a function of grain diameter for quartz spheres in water at 20°C. For comparison the predictions of fall velocity for the same system according to Stokes' Law and the impact formula are shown (after Gibbs *et al.* 1971).

Stoke's Law applies only for very small grains in which settling takes place through laminar flow.

Derivation of Stoke's Law:

- In the case of a small settling grain through a fluid, we start by assuming a low Reynold's number flow (i.e., flow around the grain is laminar, not turbulent). We also assume a spherical grain.

  – Given these assumptions, there is an experimentally-determined relationship between a sphere with radius r, settling velocity Ug, fluid viscosity m, and surface drag D, such that:

    - D=6pmrUg

  – During steady state descent through the liquid at low Reg (i.e., low grain Reynold's numbers), the surface drag force and the net buoyancy force acting on the grain must be balanced by the gravitational forces acting on the grain. That is:

    - msg = D + mfg *(where ms = grain mass, and mf = fluid mass)*

For a sinking sphere:

$$\frac{4}{3}\pi r^3 \rho_s g = 6\pi\mu r U_g + \frac{4}{3}\pi r^3 \rho_f g$$

Cancelling π on both sides:

$$\frac{4}{3} r^3 \rho_s g = 6\mu r U_g + \frac{4}{3} r^3 \rho_f g$$

Rearranging and simplifying:

$$6\mu r U_g = \frac{4}{3} r^3 \rho_s g - \frac{4}{3} r^3 \rho_f g$$

$$U_g = \frac{4r^3 g}{18\mu r}\left(\rho_s - \rho_f\right)$$

$$U_g = \frac{2r^2 g}{9\mu}\left(\rho_s - \rho_f\right)$$

Paths of grain motion

- Three general possibilities for grain motion once threshold motion has been exceeded:
- **1) Rolling:** continuous contact with the bottom
- **2) Saltating** (Latin saltare; to jump): series of ballistic hops or jumps characterized by steeply angled (>45°) ascent from the bed to height of a few grain diameters and a shallow-angled (>10°) descent path back to the bed.
- **3) Suspended motion (suspension)** in which grains move up in generally longer and more irregular trajectories higher up from the bed than saltation. Grain held in suspension by upward acceleration during turbulence.
- **4) Incipient suspension:** Upward acceleration during descending part of a saltant trajectory as turbulence starts to affect saltation.

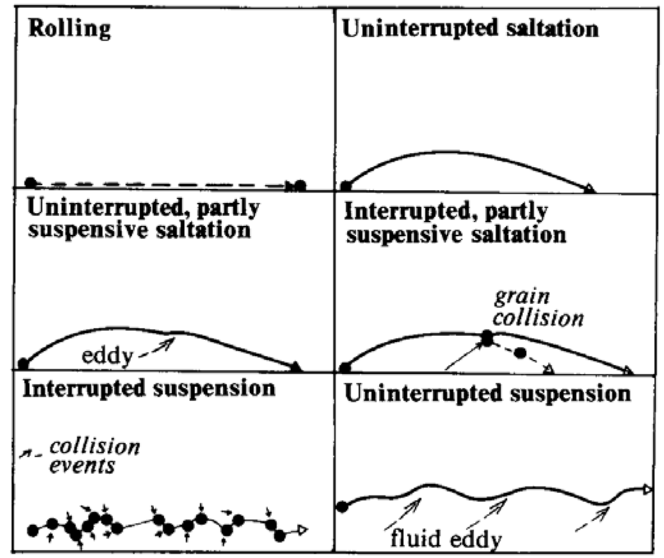


Figure 6.6 Schematic illustrations of grain paths in 'bedload' and 'suspended' load (after Leeder 1979).

Concept of transport stage

defined as the ratio of U* to U* c
(U*c = the critical boundary shear velocity needed for grain motion).
U*/U* c > 1; transport stage has been reached.
U*/U*c < 1; transport stage has not been reached.

How does transport stage affect the trajectories of moving grains?

1) The proportion of time spent by grains in each of these three modes (rolling, saltating, suspension) is a direct function of transport stage.

2) 2) Statistically, increases in length and height of saltation trajectories and incipient saltation trajectories are direct functions of transport stage.

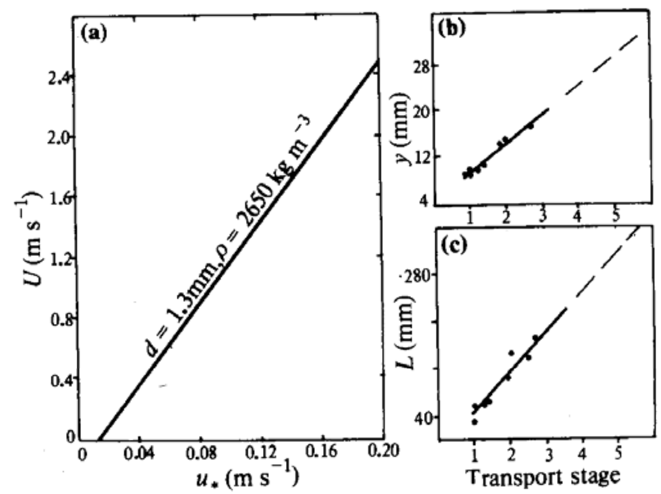


Figure 6.7 Plot of mean forward grain speed U against fluid shear velocity u_* for a 1.3 mm quartz grain (derived by Leeder 1979 from data in Abbott & Francis 1977). (b) Plot of mean maximum saltation height $\bar{y}$ against transport stage for 8.3 mm grains. (c) Plot of mean saltation length $\bar{L}$ against transport stage for 8.3 mm grains (modified after Abbott & Francis 1977). Transport stage is given by u_*/u_{*c} where u_{*c} is the critical shear velocity for initiation of motion.

- Major differences in behavior of grains in water and air result from the large density contrast (2000/1) for quartz/air relative to (1.65/1) for quartz/water.
  - In aeolian (subaerial) systems, saltating grains carry sufficient momentum to ‘splash’ up other grains. The low density of air also allows grains to reach higher maximum trajectory heights (500-600 grain diameters).
  - In water (subaqueous) systems, grains are much more restrained by buoyancy effects.
    - a) Impacting grains of up to fine gravel does not disturb other grains on the bed.
    - b) Impacted grains that do respond to the impact do not lift up immediately to begin another saltation. Linear momentum is not conserved, and it follows that the initial ascent of a saltating grain in water must be the result of Bernoulli lift forces.
- Suspension: When does the onset of full suspension occur?
  - It can be roughly estimated by assuming that the root-mean-square values for the upward vertical velocity fluctuations close to the bed begin to exceed the fall velocity of the saltating grains. $v_g{}^2$ = SQRT (v'^2)
- Experiment shows that we can approximate full suspension being reached when $v_g/U^* = 0.8$.
- All of the above equations and discussion only apply to situations where grains have no effect on each other during transport (e.g. saltation paths are not modified by grain/grain collisions.)
- Kinetic theory for collision dynamics shows that simple saltations will cease to exist over a transport stage of ~2. At this point, the grains close to the bed will begin to move as a concentrated granular dispersion dominated by grain/grain impacts.

Some important terms you should know:

- **Bedload (traction load):** includes rolling, saltating, and collision-interrupted grains. Grains in bedload will transfer momentum to the stationary bed surface by solid/solid contacts.
- **Suspension Load:** includes all grains kept aloft by turbulence. Weight force of suspended grains is balanced by upward momentum trnansfer from fluid eddies.
- **Washload and dustload:** broad term used to describe more or less permanently suspended clay-grade fines present in water and air flows, respectively.