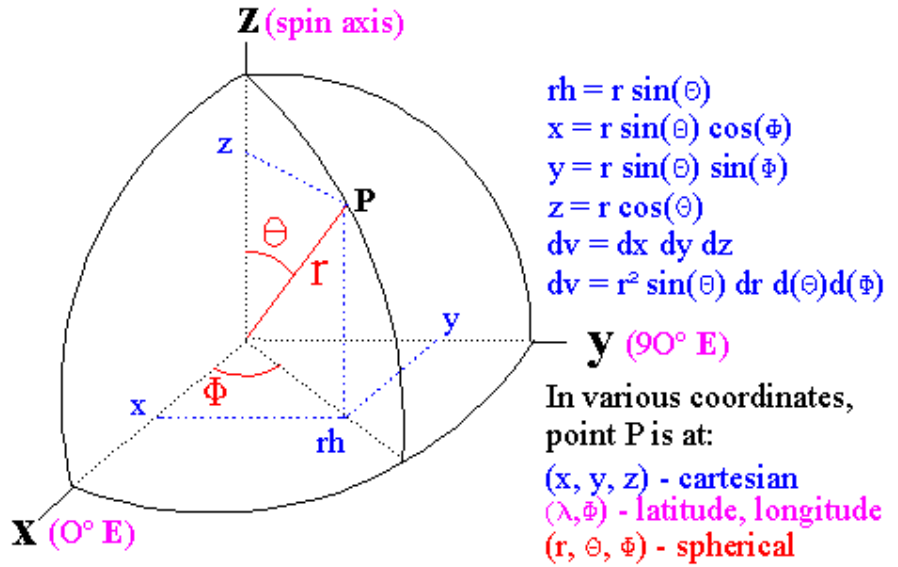


Vector **dot products** provide one way to find the distance between two points on a sphere.

Consider each of two points on a sphere as vectors defined by longitude, latitude, and radius ( $\Phi, \lambda, r$ ). The Cartesian coordinates of those points are  $x, y, z$  in the standard right-hand coordinate system. In the plane of the equator,  $x$  and  $y$  point towards  $0^\circ$  and  $90^\circ\text{E}$  longitude respectively;  $z$  is aligned with the spin axis.



The **dot product** is defined as a scalar product:

$$\vec{A} \bullet \vec{B} = |\vec{A}| |\vec{B}| \cos(\delta)$$

where  $\delta$  (delta) is the angle between the two vectors. Thus, to find the great circle angular distance between two vectors:

$$\delta = \cos^{-1} \left( \frac{\vec{A} \bullet \vec{B}}{|\vec{A}| |\vec{B}|} \right)$$

So, how do we evaluate this expression? By definition:

$$\delta = \cos^{-1} \left( \frac{A_x B_x + A_y B_y + A_z B_z}{|\vec{A}| |\vec{B}|} \right)$$

If we consider the vectors to be unit vectors, here the unit would be Earth's radius, then:

$$\delta = \cos^{-1} (A_x B_x + A_y B_y + A_z B_z).$$

Thus, find the  $x, y,$  and  $z$  coordinates of  $(\Phi, \lambda, r)$  and evaluate the above expression. As an example, find the distance between Missoula and Rabaul, Papua New Guinea:

What is the great circle distance from Missoula to Rabaul, Papua New Guinea?		
Missoula: $(\phi, \lambda) =$ (246°E, 46°N)	Rabaul: $(\phi, \lambda) =$ (152°E, 4°S)	<b>The products</b>
$A_x = -.283$	$B_x = -.881$	$A_x B_x = 0.249$
$A_y = -.635$	$B_y = 0.468$	$A_y B_y = -.297$
$A_z = 0.719$	$B_z = -.070$	$A_z B_z = -.050$
		<b>SUM = -.098</b>

$$\delta_{MSO-RAB} = \cos^{-1}(A_x B_x + A_y B_y + A_z B_z)$$

$$\delta_{MSO-RAB} = \cos^{-1}(-.098)$$

$$\delta_{MSO-RAB} = 95.6^\circ$$

$$95.6^\circ * \frac{2 * \pi * 6,371km}{360^\circ} = 10,630km,$$

(if Earth was spherical)