For continental lithosphere the general equation for an elastic plate with a surface load qa(x), end load P, and hydrostatic restoring force is:

$$D\cdot\frac{d^4}{dx^4}w(x) + P\cdot\frac{d^2}{dx^2}w(x) + (\rho m - \rho c)\cdot g\cdot w(x) = qa(x)$$

Now we'll assume a periodic load of a particular wavelength, λ:

$$h(x) = h0\cdot\sin\left(2\cdot\pi\cdot\frac{x}{\lambda}\right)$$

so that the load on the lithosphere becomes:

$$qa(x) = g\cdot\rho c\cdot h0\cdot\sin\left(2\cdot\pi\cdot\frac{x}{\lambda}\right)$$

which we will substitute into the general equation. Then we also let the end load, P, equal zero yielding the equation for an elastic plate with a periodic surface load and hydrostatic restoring force:

$$D\cdot\frac{d^4}{dx^4}w(x) + (\rho m - \rho c)\cdot g\cdot w(x) = g\cdot\rho c\cdot h0\cdot\sin\left(2\cdot\pi\cdot\frac{x}{\lambda}\right)$$

This, above, is the equation we want to solve and a solution is:

$$w(x) = w0\cdot\sin\left(2\cdot\pi\cdot\frac{x}{\lambda}\right)$$

Its fourth derivative is:

$$\frac{d^4}{dx^4}w(x) = w0\cdot\sin\left(2\cdot\pi\cdot\frac{x}{\lambda}\right)\cdot\left(2\cdot\frac{\pi}{\lambda}\right)^4$$

If we now plug both w(x) and the fourth derivative into our equation, we get:

$$D\cdot w0\cdot\sin\left(2\cdot\pi\cdot\frac{x}{\lambda}\right)\cdot\left(2\cdot\frac{\pi}{\lambda}\right)^4 + (\rho m - \rho c)\cdot g\cdot\left(w0\cdot\sin\left(2\cdot\pi\cdot\frac{x}{\lambda}\right)\right) = g\cdot\rho c\cdot h0\cdot\sin\left(2\cdot\pi\cdot\frac{x}{\lambda}\right)$$

Divide through by the sine term and pull w0 out front and you get:

$$w0\cdot\left[D\cdot\left(2\cdot\frac{\pi}{\lambda}\right)^4 + (\rho m - \rho c)\cdot g\right] = g\cdot\rho c\cdot h0$$

which you can solve for w0 which gives the amplitude of deflection w0 as a function of the amplitude of load, its wavelength, and the various geologic parameters:

$$w0 = \frac{h0}{\frac{D}{\rho c\cdot g}\left(2\cdot\frac{\pi}{\lambda}\right)^4 + \frac{\rho m}{\rho c} - 1}$$

thus:

$$w(x) = \frac{h0}{\frac{D}{\rho c\cdot g}\left(2\cdot\frac{\pi}{\lambda}\right)^4 + \frac{\rho m}{\rho c} - 1}\cdot\sin\left(2\cdot\pi\cdot\frac{x}{\lambda}\right)$$

Now that we have the equation for the amplitude of deflection for a periodic load of a specified wavelength, let's look at it for topographic loads of two different wavelengths; let all other parameters remain constant:

$\rho m := 3250 \quad \rho c := 2670 \quad g := 9.8 \quad E := 7 \cdot 10^{10} \quad v := 0.25 \quad G := 6.673 \cdot 10^{-11} \quad bm := 35000 \quad ho := 1000$

$Te := 6000 \quad D := \dfrac{E \cdot Te^3}{12 \cdot (1 - v^2)} \quad D = 1.344 \cdot 10^{21} \qquad \lambda 1 := 500000 \quad \lambda 2 := 100000 \quad x := 0, 500 .. \lambda 1$

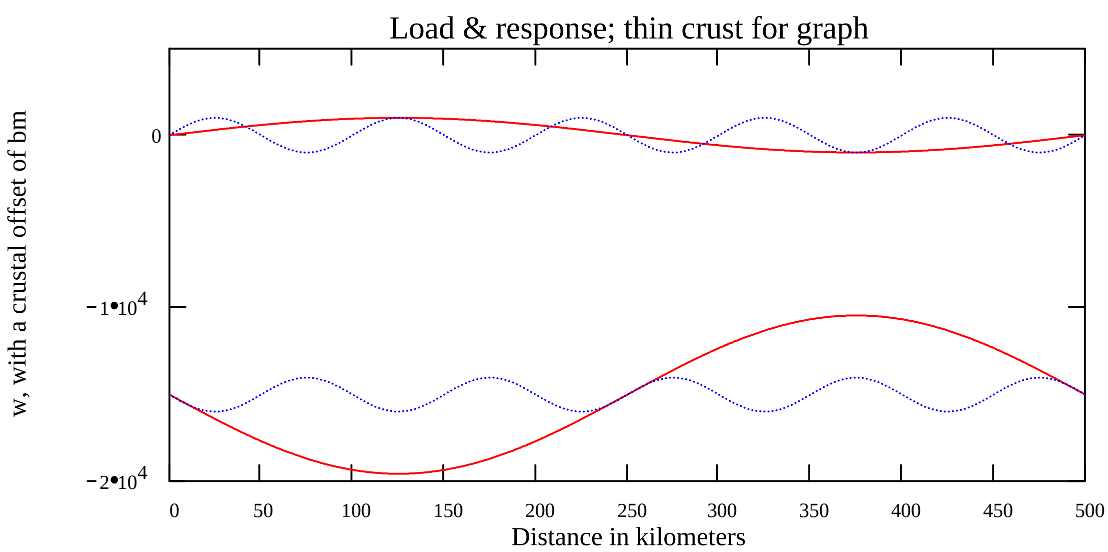
The deflection of the lithosphere w(x) is due to the loading of the topography (h(x)):

Case #1, 500km, solid lines:

$$w1(x) := \frac{-ho}{\left[\dfrac{D}{\rho c \cdot g} \cdot \left(\dfrac{2 \cdot \pi}{\lambda 1}\right)^4 + \dfrac{\rho m}{\rho c} - 1\right]} \cdot \sin\left(\frac{2 \cdot \pi \cdot x}{\lambda 1}\right)$$

Case #2, 100km, dotted lines:

$$w2(x) := \frac{-ho}{\left[\dfrac{D}{\rho c \cdot g} \cdot \left(\dfrac{2 \cdot \pi}{\lambda 2}\right)^4 + \dfrac{\rho m}{\rho c} - 1\right]} \cdot \sin\left(\frac{2 \cdot \pi \cdot x}{\lambda 2}\right)$$



Given a periodic load we found an equation for the flexure. Another way to think about this is via linear filters. Consider the input to the system to be the load and the output to be the lithosphere's flexural response. In this case, the filter relating input and output is the flexural response function which we can solve for:

Input *{flexural response function} = Output
or
Load * {flexural response function} = Flexure (i.e., w(x))
thus
flexural response function = Flexure/Load

$$\frac{Flexure}{Load} = \frac{w(x)}{g \cdot \rho c \cdot h0 \cdot \sin\left(2 \cdot \pi \cdot \dfrac{x}{\lambda}\right)}$$

$$\frac{Flexure}{Load} = \frac{\dfrac{h0}{\dfrac{D}{\rho c \cdot g}\left(2\cdot\dfrac{\pi}{\lambda}\right)^4 + \dfrac{\rho m}{\rho c} - 1}\cdot \sin\left(2\cdot\pi\cdot\dfrac{x}{\lambda}\right)}{g\cdot\rho c\cdot h0\cdot\sin\left(2\cdot\pi\cdot\dfrac{x}{\lambda}\right)}$$

$$\frac{Flexure}{Load} = \frac{\dfrac{1}{\dfrac{D}{\rho c \cdot g}\left(2\cdot\dfrac{\pi}{\lambda}\right)^4 + \dfrac{\rho m}{\rho c} - 1}}{g\cdot\rho c}$$

$$\frac{Flexure}{Load} = \frac{1}{D\left(2\cdot\dfrac{\pi}{\lambda}\right)^4 + (\rho m - \rho c)\cdot g} = Flexural\_Response\_Function = FRP$$

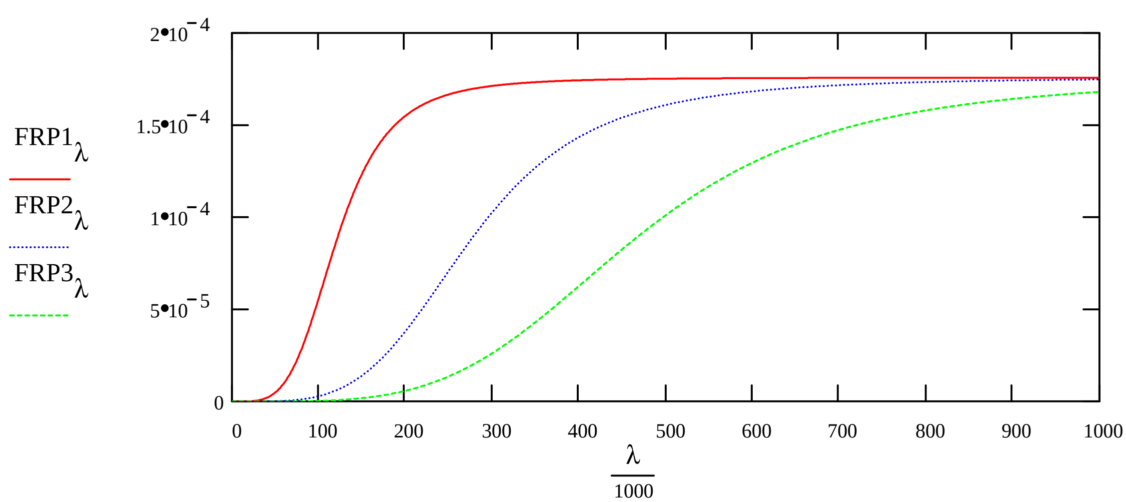
Now look at the Flexural Response Function for values of elastic thickness equal to 5 km, 15 km, and 30km:

$\lambda := 1, 1000 .. 1000000$ thus, wavelengths go from 1 to 1000 kilometers

Te = 5 km $Te1 := 5000$ $D1 := \dfrac{E\cdot Te1^3}{12\cdot(1 - v^2)}$ $FRP1_\lambda := \dfrac{1}{D1\left(2\cdot\dfrac{\pi}{\lambda}\right)^4 + (\rho m - \rho c)\cdot g}$

Te = 15 km $Te2 := 15000$ $D2 := \dfrac{E\cdot Te2^3}{12\cdot(1 - v^2)}$ $FRP2_\lambda := \dfrac{1}{D2\left(2\cdot\dfrac{\pi}{\lambda}\right)^4 + (\rho m - \rho c)\cdot g}$

Te = 30 km $Te3 := 30000$ $D3 := \dfrac{E\cdot Te3^3}{12\cdot(1 - v^2)}$ $FRP3_\lambda := \dfrac{1}{D3\left(2\cdot\dfrac{\pi}{\lambda}\right)^4 + (\rho m - \rho c)\cdot g}$



At short wavelengths there is no flexural response, the strength of the lithosphere supports the load and the corresponding Bouguer anomalies would be near zero. At the longest wavelengths the plate is weak, behaving like an Airy-type structure, and the response for the different elastic thicknesses approaches the same value; Bouguer anomalies would be large and negative. At intermediate wavelengths the response is flexural and you can see a clear, diagnostic difference between the weak (Te = 5 km), intermediate (Te = 15 km), and strong (Te = 30 km) plates.

In the literature, you'll more commonly see the flexural response function plotted versus the log of the wavenumber where wavenumber is just {2*π/λ}.

$$k_{\lambda} := 2 \cdot \frac{\pi \cdot 1000}{\lambda}$$