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Earth Models Obtained by Monte Carlo Inversion

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The problem of uniqueness of earth structures obtained by the inversion of geophysical data is still unsolved. Monte Carlo methods offer the advantage of exploring the range of possible solutions and indicate the degree of uniqueness achievable with currently available geophysical data. This procedure was applied to the earth by using the following data: 97 eigenperiods, travel times of compressional and shear waves, and mass and moment of inertia of the earth. Indirect use was made of $dt/d\Delta$ data obtained from arrays. Five million models have been examined, and six have passed all tests. Results are as follows: (1) The earth's core is inhomogeneous, consistent with Fe(15–25%)-Si for the outer core and Fe(20–50%)-Ni alloy for the inner core. (2) The radius of the core is increased by 5 to 20 kilometers. (3) Large density and velocity gradients are found in the transition region without prior assumption of an equation of state. The transition region is a compositional boundary as well as a zone in which phase changes occur. The lower mantle shows an increase of the FeO/MgO + FeO ratio by a factor of 2 compared with the upper mantle. This could inhibit mantle-wide convection. (4) Upper mantle models fit better than 'standard' models when they are more complex, showing large fluctuations in shear velocity and density. Such complexity might be expected if the mantle is chemically and mineralogically zoned, if there are high thermal gradients, and if partial melting takes place. However, the magnitude of the fluctuations suggests that the zoning is lateral in nature and that the mantle is variable in composition laterally ranging from pyrolite to eclogite.

INTRODUCTION

In recent years a number of earth models based on one or more of the following have been proposed: (1) seismic body-wave travel times, (2) dispersion of surface waves and/or periods of the earth's free oscillations, (3) mass and moment of inertia of the earth, (4) an equation of state, either empirical or theoretical relating seismic velocities and densities, and (5) petrological-geochemical arguments. Items 1, 2, and 3 above are basic data which must constrain all models. These data suffer in that they are incomplete and uncertain owing to observational errors, lateral variations in the crust and mantle, and splitting of spectral lines. Aside from the unsolved formal problem of uniqueness of inversions of geophysical data, the many models proposed from this set of data provide empirical evidence for the lack of uniqueness. Items 4 and 5 have often been used to add additional constraints to solutions, but, unfortunately, many assumptions involved in this procedure are difficult to evaluate.

In this paper we describe a Monte Carlo method for generating large numbers of models, selecting only those which satisfy the first three items above. The procedure offers the advantage of finding models without bias stemming from preconceived or oversimplified notions of earth structure. A measure of uniqueness of the solutions derived from currently available data is available to the degree to which the successful models agree or disagree. The successful models are analyzed and are interpreted in terms of variations in phase and composition in the mantle and core as implied by laboratory experiments on iron-magnesium oxides and silicates, and iron-silicon, iron-nickel alloys. Although Monte Carlo inversion has been used before (see for example *Keilis-Borok and Yanovskaya* [1967] for a review of work in the USSR; *Press and Biehler* [1964]), the present procedure is the most comprehensive we known of both in the amount of data used and in the depth range covered. A discussion of our successful density solutions was presented earlier [*Press*, 1968].

METHOD

The method is best described by the computer flow diagram in Figure 1. An algorithm is used to generate random numbers in the

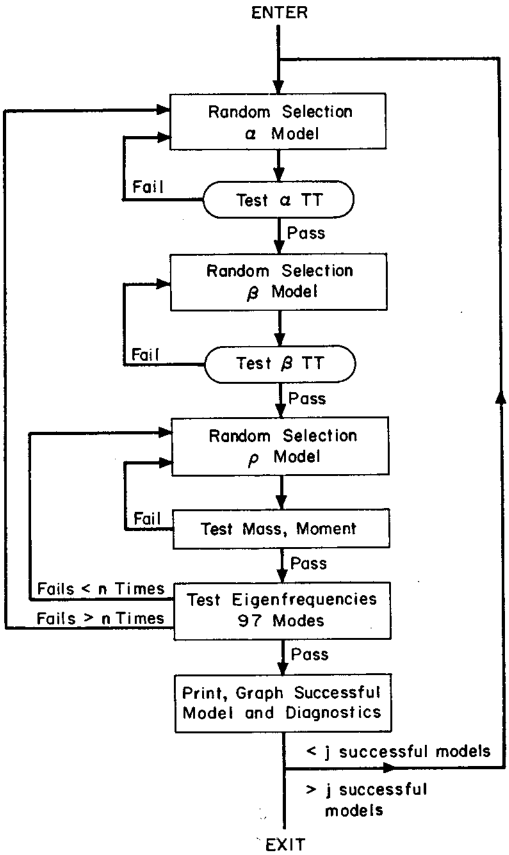

Fig. 1. Flow diagram of Monte-Carlo inversion procedure.

computer, which in turn are used to select the random compressional (α), shear (β), and density (ρ) values that characterize a given model. Actually, the selection is prescribed to fall within upper and lower bounds that can be varied as input constants. The ranges permitted for the randomly varying parameters are broader than the bounds formed by most previously suggested models.

Travel times are computed for the compressional and shear velocity distributions by using subroutines based on *Bullen's* [1961] method in which the earth is treated as a multilayered sphere, the velocity varying according to a power law within each layer. The theoretical travel times are compared with recently observed travel times for *P* waves [*Carder et al.*, 1966] and *S* waves [*Doyle and Hales*, 1967; *Kogan*, 1960]. The upper and lower bounds and the experimental travel-time tables are input constants to the program. Similarly, the mass M and moment of inertia I are computed for the density distribution, and the model values are compared with the most recent experimental determinations: $M = 5.976 \times 10^{27}$ grams, $I/Ma^2 = 0.3308$, a is the equatorial radius. If any of these three tests fails, the program enters a loop in which the corresponding parameter is selected repeatedly until a model tests successfully. The program contains a feature whereby the gradients of α, β, ρ distributions are restricted to a maximum of l reversals of sign, where l is an input constant (typically 6). Where justified, this can be used to restrict the complexity of models.

If an α, β, ρ distribution is found by this procedure, it is then tested against the free oscillations of the earth. The experimental eigenperiods (summarized by *Pekeris* [1966] and *Nowroozi* [1966]) are entered as input constants. The model eigenperiods are computed by using variational parameters of eigenperiod with respect to α, β, ρ. Approximately 25,000 variational parameters are entered into the computer from magnetic tape during the input phase of the program. This procedure makes possible a rapid and accurate computation of eigenperiods by table lookup and is the essential feature of the method. Without it Monte Carlo techniques would be prohibitive in computer costs because of their inherent inefficiency. The variational parameters were computed by *Wiggins* [1968] with respect to a standard model proposed by *Birch* [1964]. Our procedure is such that the standard model does not introduce bias in the selection of random models. Eigenperiods computed in this way were tested against exact computations and were found to be sufficiently accurate for our purposes.

In the eigenperiod test all but m modes must be predicted successfully, m being assigned to take account of the possibility of misidentified modes. If the eigenperiod test fails p times, the program enters a loop involving reselection of densities only. After p failures of the eigenperiod test, the program discards the α, β, ρ distributions and selects new ones. This procedure improves the speed with which successful models can be found, since the ρ selection-testing procedure is more efficient than that for α and β. Typical values of p were 5–10.

Some additional features of the program and procedures used are as follows:

1. The radius of the core is selected randomly for each model in the range 3473 ± 25 km.

2. In computing travel times, mass, moment of inertia, and eigenperiods, values of α, β, ρ were specified at 88 points within the earth. Although all of the point values could be randomly selected, we chose the time-saving device of randomly varying only 23 points, and obtaining the remaining values by linear interpolation.

3. Superadiabatic density gradients were not permitted in the core, which was assumed to be fluid throughout.

4. The compressional velocity in the core was fixed according to Birch's model.

5. Output included graphs and tables of α, β, ρ, seismic ratio ϕ, and mean atomic weight m for the successful models, graph, and tables of the deviations between observed and computed mass, moment of inertia, travel time, and eigenperiods for successful models, diagnostics of number of models tried, and reasons for failure of unsuccessful models.

6. Between 200,000 and 250,000 models could be generated and tested per hour (the test being carried as far as the first failure) on an IBM 360/65 computer.

7. Eigenperiods tested were ${}_0S_2$–${}_0S_{48}$, ${}_0T_2$–${}_0T_{44}$, ${}_0S_0$, ${}_1S_2$–${}_1S_5$, ${}_1S_7$, ${}_1S_8$, ${}_1S_{11}$, ${}_2S_7$, and successful models were required to predict ${}_0S_2$, ${}_0S_0$, ${}_0T_2$ and 84 of the remaining 94 modes within assigned tolerances. Actually, because of the overlapping information content in the modes, specifying ten permissible failures led to models with less than five failures.

8. Body waves tested were *P*, *PcP*, *S*, *ScS*, at a prescribed number of distances (usually 6) in the range 25°–100°. Failure at any of these distances resulted in rejection of the model.

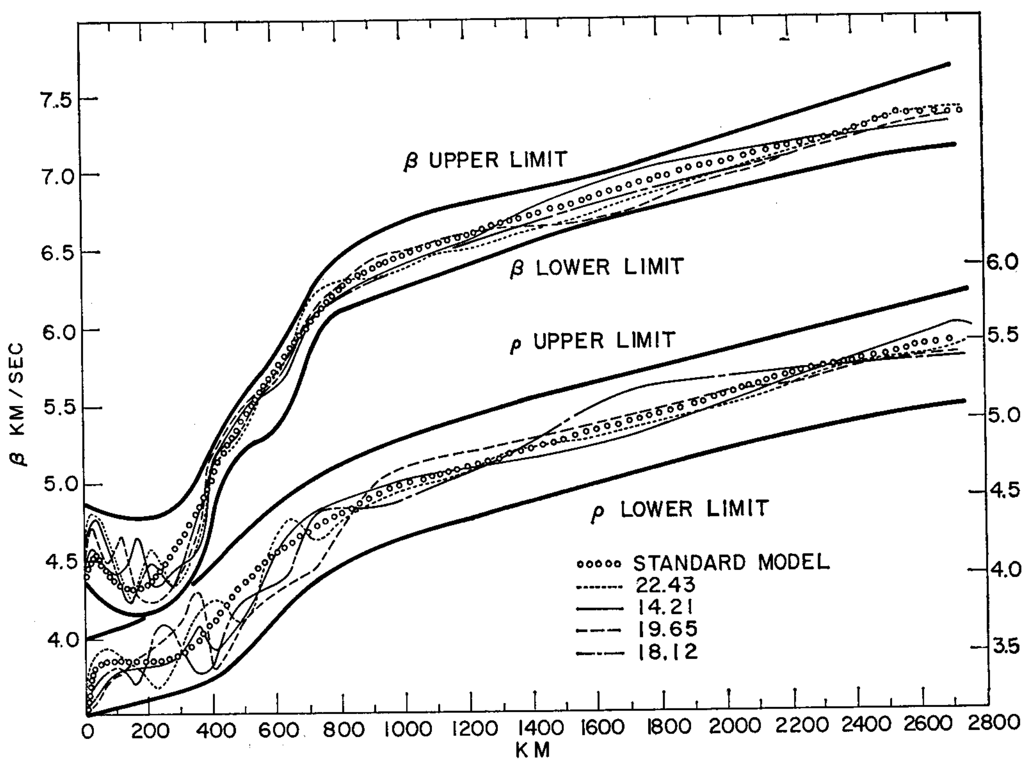

Fig. 2. Shear velocity (β) and density (ρ) distributions for the mantle for the standard model and for three successful Monte Carlo models. Model number also gives increase in core radius. Permissible β, ρ region shown by heavy curves. Model 14.21 failed ${}_0S_2$ test but is of interest because of excellent fit of higher modes.