## Cooling a Dike by Conduction;Difference Equations

$\frac{d}{dt}\cdot f(x,t) = diff\cdot\frac{d^2}{dx^2}\cdot f(x,t)$ is the 1D, time-dependent heat conduction equation; diff is diffusivity.

We will approximate the 1D heat flow equation with a finite-difference equation.

$t := 0, 1 .. 200$ . . . will be the time steps for the iterative solution for the static graph.

$x := 1 .. 50$ . . . x is distance coordinate in meters

The density, specific heat, and thermal conductivity are:

$\rho := 2800$ (Kg/m$^3$) $Cp := 750$ (J/kg $^o$K) $k := 2$ W/(M$^o$C)

$diff := \frac{k}{\rho\cdot Cp}$ diff is diffusivity which is m$^2$/s (a watt is a Joule/sec)

$diff := diff\cdot 365\cdot 24\cdot 60\cdot 60$ $diff = 30.034$ now in m$^2$/year.

$\alpha := diff\cdot\frac{3}{365}$ . . . $\alpha$, the diffusion constant, needs to be << 0.50 for stability

$\alpha = 0.247$ and **time-steps are 3 days each**.

$Dike := 20, 21 .. 30$ ... the x distance range where the dike is.

Initial conditions: time t=0

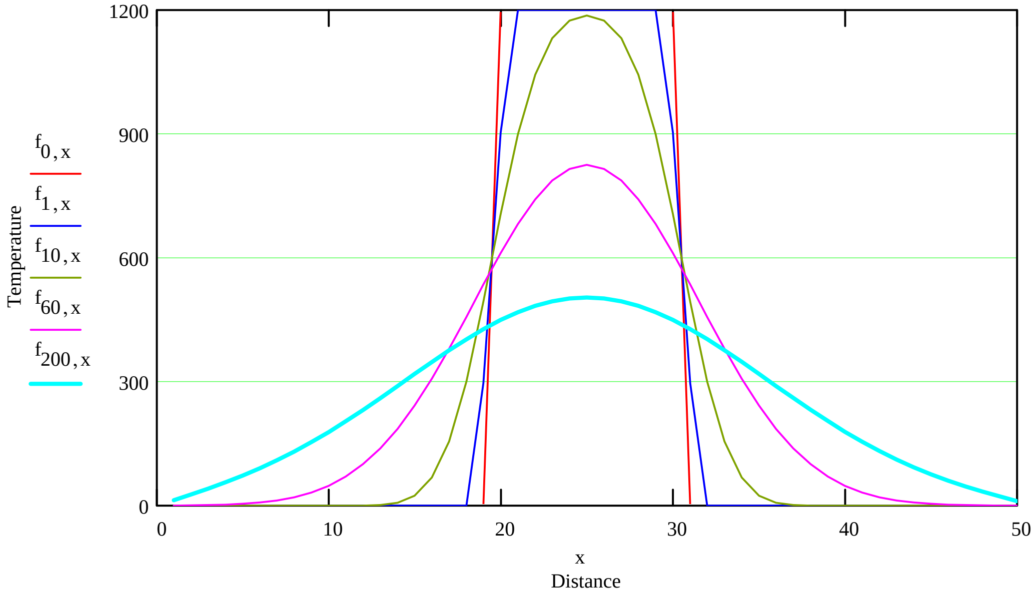
$f_{0,x} := 0$ $f_{0,0} := 0$ $f_{0,51} := 0$ . . . T is 0 everywhere *except* where the dike is

$f_{0,Dike} := 1200$ . . . 1200 $^o$C emplacement temperature

Difference equation for diffusion:

$f_{t+1,x} := f_{t,x} + \alpha\cdot\left(f_{t,x-1} - 2\cdot f_{t,x} + f_{t,x+1}\right)$ compare to: $\frac{d}{dt}\cdot f(x,t) = diff\cdot\frac{d^2}{dx^2}\cdot f(x,t)$

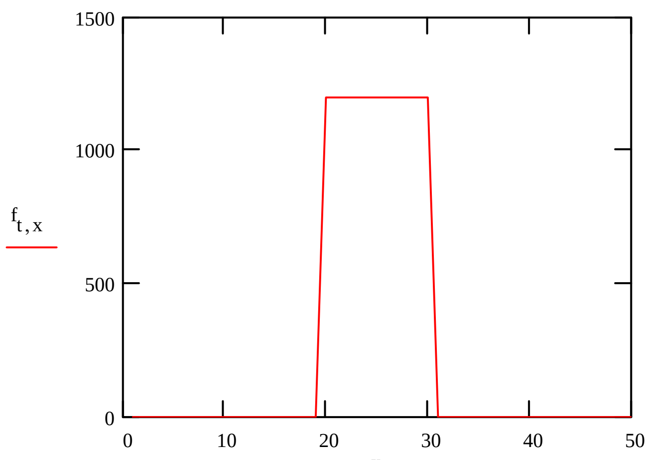
In the difference equation we are just looking at each time step and using a numerical approximation for the second derivative ($f_{t,x-1} - 2\cdot f_{t,x} + f_{t,x+1}$).



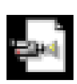

Now animate the propagating isotherms:

$t := 0, 1 .. FRAME$ . . . will be the time steps for the iterative solution in the animation

$f_{t+1,x} := f_{t,x} + \alpha \cdot \left(f_{t,x-1} - 2 \cdot f_{t,x} + f_{t,x+1}\right)$ ... to include equation in animation window.




Sometimes Mathcad's animation works sometimes it doesn't. Once, when I got it to work it produced:


Video Clip