

Suppose we have a crustal model with exponential decrease in heat producing elements in the upper 10 kilometers and constant heat production below that. First we need a couple constants and a range of depths to calculate the geotherms;  $A_0$  is heat production at the surface,  $k$  is thermal conductivity,  $q_b$  is heat flow from the mantle, the crust will be 35 km thick:

$$A_0 := 2.5 \cdot 10^{-6} \frac{\text{watt}}{\text{m}^3} \quad k := 2 \frac{\text{watt-deg}}{\text{m}} \quad z := 0 \cdot \text{km}, 1 \cdot \text{km} \dots 35 \cdot \text{km} \quad q_b := .02 \frac{\text{watt}}{\text{m}^2}$$

$$D := 10 \cdot \text{km} \quad H := 10 \cdot \text{km} \quad A(z) := A_0 \cdot e^{-\frac{z}{D}} \quad A(10 \cdot \text{km}) = 9.197 \cdot 10^{-7} \frac{\text{watt}}{\text{m}^3}$$

First, assuming a steady state situation, calculate the heat flow out of the surface;  $q(\text{surf}) = q_b + q(\text{slab}) + q(\text{exp})$ . Heat flow from the top of the 10 km slab with constant  $A=A(10 \text{ km})$  is  $q_{\text{slab}}$ :

$$q_{\text{slab}} := 25 \cdot \text{km} \cdot A(10 \cdot \text{km}) \quad q_{\text{slab}} = 0.023 \frac{\text{watt}}{\text{m}^2}$$

Heat flow from the section with exponentially decreasing heat production is  $q_{\text{exp}}$ :

$$q_{\text{exp}} := A_0 \cdot D \cdot \left( 1 - e^{-\frac{H}{D}} \right)$$

So the total heat flow at the surface is  $q_0$ :

$$q_0 := q_b + q_{\text{slab}} + q_{\text{exp}} \quad q_0 = 0.059 \frac{\text{watt}}{\text{m}^2}$$

Next, calculate the temperature at the base of the 10 km thick zone that has exponentially decreasing heat production with depth.

$$z := 0 \cdot \text{km}, 1 \cdot \text{km} \dots 10 \cdot \text{km} \quad T_e(z) := \left[ \frac{A_0 \cdot D}{k} \cdot \left( D - D \cdot e^{-\frac{z}{D}} - z \cdot e^{-\frac{H}{D}} \right) + \frac{q_b + q_{\text{slab}}}{k} \cdot z \right] \cdot \text{deg} \quad T_e(10 \cdot \text{km}) = 247.992$$

Now calculate the temperature profile in a 25 km slab beneath the exponential section. We'll use the heat production at 10 km for the constant heat production value throughout the remainder of the crust.

$$H := 25 \cdot \text{km} \quad z_s := 0 \cdot \text{km}, 1 \cdot \text{km} \dots 25 \cdot \text{km} \quad T_s(z) := T_e(10 \cdot \text{km}) + \left[ \frac{A(10 \cdot \text{km})}{k} \cdot \left( H \cdot z - \frac{z^2}{2} \right) + \frac{q_b}{k} \cdot z \right] \cdot \text{deg}$$

For the whole crust, use a LOGICAL IF statement to combine the graphs for  $T_s(z)$  and  $T_e(z)$ :

$$T_{\text{tot}}(z) := \text{if}(z \leq 10 \cdot \text{km}, T_e(z), T_s(z - 10 \cdot \text{km})) \quad \text{Now fudge some things to get the graph units correct:}$$

$$z := 0 \cdot \text{km}, 1 \cdot \text{km} \dots 35 \cdot \text{km}$$

