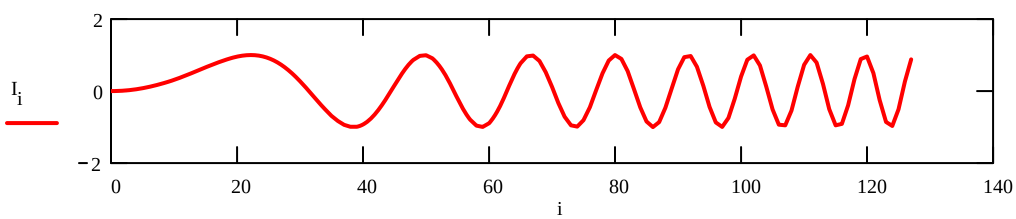
Here is an example of convolution and correlation. Convolution is used to make a synthetic seismogram. The input signal is long and changes frequency with time, this is similar to a vibroseis signal. After convolving the input signal with a series of reflection coefficients, correlation with the input signal is used to recover the series of reflection coefficients.

First define an input signal whose frequency increases as a function of time:

$n := 127$ $\qquad$ $i := 0 .. n$ $\qquad$ $I_i := \sin\left(\frac{.01}{\pi} \cdot i \cdot i\right)$




We also need a stratigraphic section represented as a series of reflection coefficients. Let those reflection coefficients be represented by $r_m$'s and add the length of the input signal to the length of the vector of reflection coefficients to make room for the convolution:

$m := 1023$ $\qquad$ $mn := m + n$

$r_0 := 0$ $\quad$ $r_{300} := -1$ $\quad$ $r_{129} := 1$ $\quad$ $r_{600} := -1$ $\quad$ $r_{664} := 1$ $\quad$ $r_{850} := 1$ $\quad$ $r_{914} := 1$ $\quad$ $r_{mn} := 0$

Now do the convolution. There will be m + n elements in the synthetic seismogram.

$k := n, n+1 .. mn$

$j := 0, 1 .. mn$

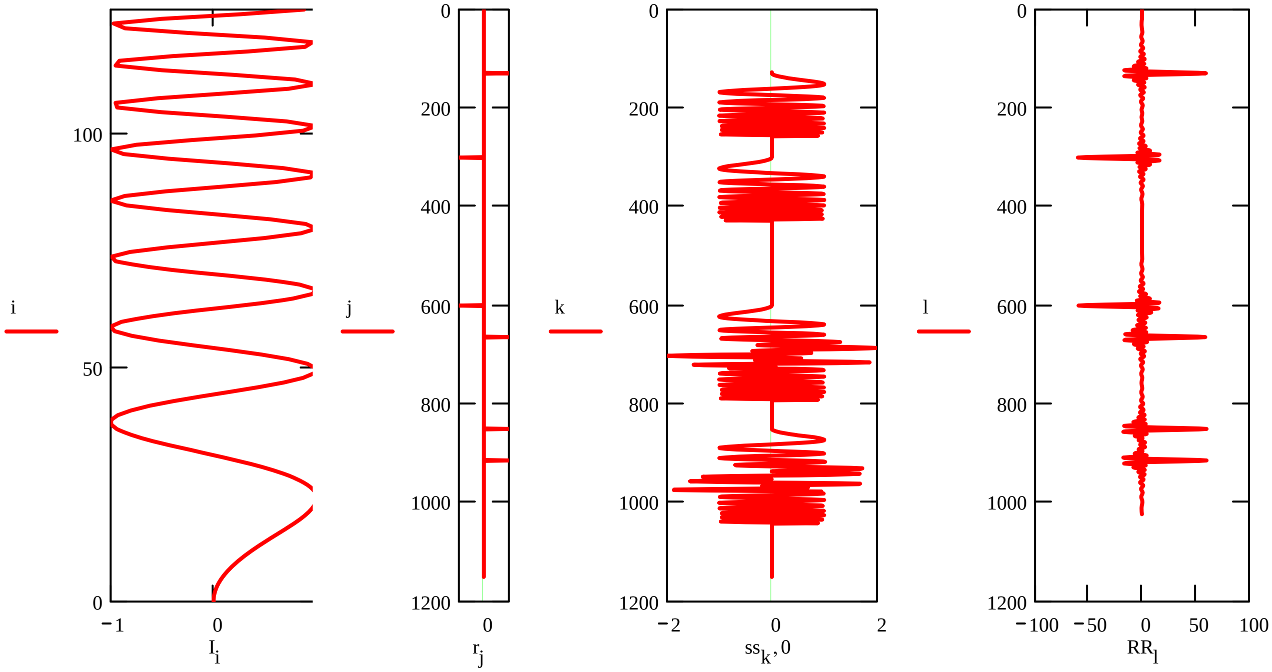
$$ss_k := \left[\sum_{n=0}^{127} \left(r_{k-n} \cdot I_n\right)\right]$$

$SS_k$ now holds the data for the synthetic seismogram. $RR_l$ will hold the data for the recovered (after cross-correlation) reflection coefficients.

$l := 0, 1 .. m$

$$RR_l := \left[\sum_{n=0}^{127} \left(ss_{l+n} \cdot I_n\right)\right]$$

Input Wave: | Reflection Coefficients: | Synthetic Seismogram: | Recovered reflection coefficients:




The first two reflections in the synthetic seismogram demonstrate that a change in the sign of the reflection coefficient changes the polarity of the resulting wavelet. Reflections three and four show superposition from reflection coefficients of opposite sign. The last two reflections are spaced the same as the previous two but have the same sign.

The amplitudes of the recovered reflection coefficients in the right-hand graph should be normalized back to unity. However, as they are, they demonstrate the efficiency of the correlation technique.

Now we can simulate some noise in the system by adding a random number to each point in the convolution. Start with the same reflection coefficients as before:

$r_0 := 0 \quad r_{300} := -1 \quad r_{129} := 1 \quad r_{600} := -1 \quad r_{664} := 1 \quad r_{850} := 1 \quad r_{914} := 1 \quad r_{mn} := 0$

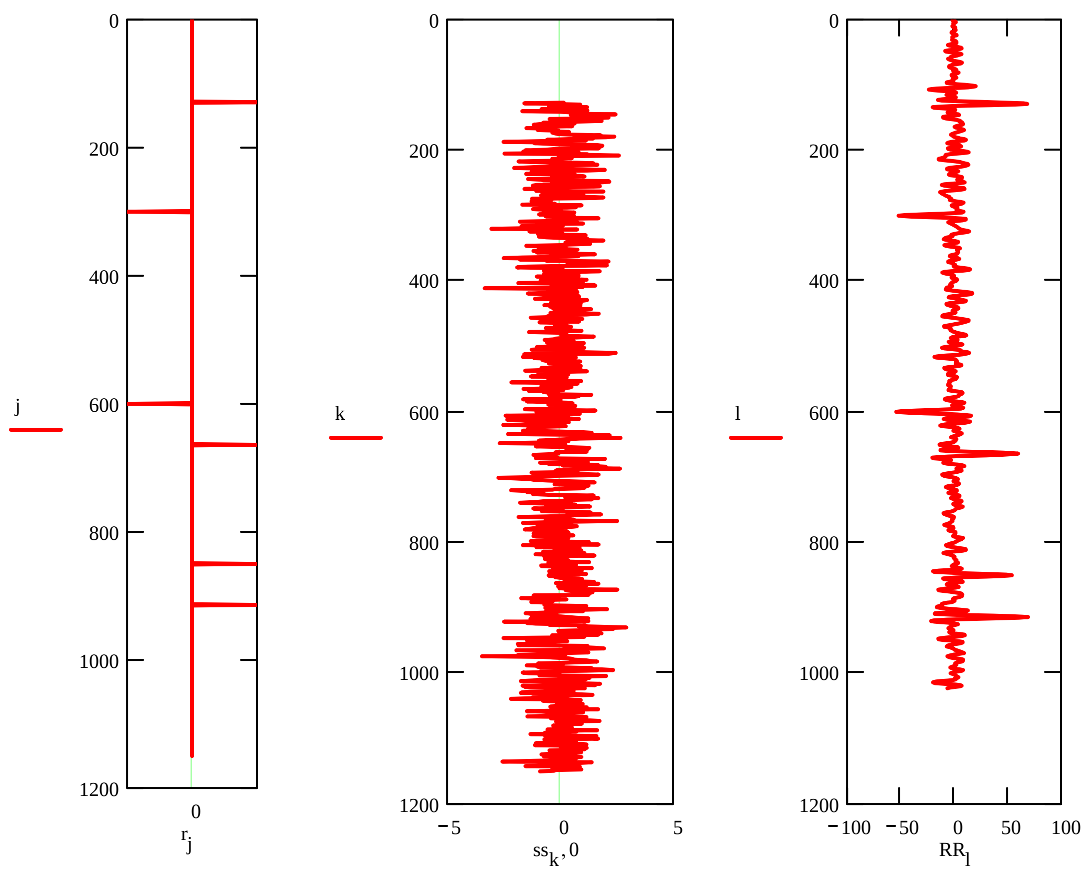
Convolution of input wave and reflection coefficients:

$$ss_k := \left[ \sum_{n=0}^{127} \left( r_{k-n} \cdot I_n + \text{rnd}(.25) - .125 \right) \right]$$

Cross-correlation of the synthetic seismogram with the input wave:

$$RR_l := \left[ \sum_{n=0}^{127} \left( ss_{l+n} \cdot I_n \right) \right]$$

Reflection Coefficients: | Synthetic Seismogram: | Recovered reflection coefficients:




Compare this synthetic seismogram with the previous one; they have the same reflection coefficients but this one has some random noise added in. Despite burying a bunch of random noise in the reflection record, the cross-correlation recovers the reflectors pretty well.