

**SPREADSHEET MODELING OF  
ELECTRICAL SOUNDING EXPERIMENTS**

**Steven D. Sheriff**

**Department of Geology  
University of Montana  
Missoula, Montana 59812**

## ABSTRACT

**Electrical resistivity sounding is useful for solving a variety of ground-water problems. To model apparent resistivities for a specific model requires one to evaluate a convolution integral. This calculation, using linear filtering, is rapid and convenient with a commercial spreadsheet. In comparison to traditional programming languages, the high-level language and built in screen-graphics provided by the spreadsheet save much programming time.**

## INTRODUCTION

Electrical resistivity sounding detects changes in the resistivity of the earth beneath a given surface location. Sounding is regularly used to solve a wide variety of ground-water problems. Heigold et al. (1979) outline a procedure for using electrical sounding to evaluate the water-producing capabilities of granular aquifers. Underwood et al. (1984) attribute significant savings of time and money to electrical sounding in siting a new municipal well. Huntley (1986), noting that electrical sounding measures a volume of sediment similar to that of an aquifer test, contributes to quantifying the relation between resistivity and permeability. Urish (1983) uses sounding to determine the most appropriate distribution of electrodes to detect and trace pollution. White (1988) uses sounding to measure the velocity of a saline tracer. Urish and Frohlich (1990) use sounding for ground-water exploration in coastal regions. Electrical sounding is an inexpensive, fast and non-invasive technique that yields useful information about subsurface conditions. This paper presents a convenient, quick and graphical way to calculate the expected apparent resistivity from sounding over any horizontally layered model of the subsurface.

Electrical resistivity soundings introduce a direct current into the ground via two electrodes. A

second pair of electrodes between the current electrodes measures the electrical potential. The apparent resistivity, calculated from these measurements at the surface, is a function of subsurface layering. Wide electrode separations allow deep layers to affect the surface measurement. Three arrangements of electrodes are in common use. The Wenner array uses equally spaced electrodes. The dipole-dipole array uses widely separated pairs of closely spaced current and potential electrodes. The Schlumberger array uses closely spaced potential electrodes and widely spaced current electrodes. The Schlumberger arrangement is common in ground-water studies and serves as the example in the rest of this paper. The spreadsheet programming techniques are readily adapted to any of the electrode configurations.

There are two approaches to using results from electrical sounding. The forward solution consists of assuming a geologic cross section or model and calculating the apparent resistivities expected from the given layering and distribution of electrodes. Forward modeling allows one to exploit information of variable value from the local area and from one's experience. For example, one might use available well control to formulate a model, calculate the expected results, then design an efficient field program to test the hypothesis. Alternatively, one could iteratively adjust a geologic model until the theoretical results fit existing field measurements. Programming a spreadsheet, with its immediate access to good graphics, to do the calculations makes the forward approach fast and convenient.

Inverse methods, like those involving least squares or linear filtering (Ghosh 1971a; Basokur, 1990), find a model directly from observed data. Inverse methods can provide a good start if one has electrical soundings from hydrologically or geologically unknown areas. However, inverse approaches do not yet include an important but fuzzy parameter. The missing characteristic is that

subsurface models must be geologically reasonable. Automated forward modelling (e.g., Zhody, 1989) partially bridges the gap between forward and inverse methods but also lacks constraint from local geologic knowledge.

The method for solving the one-dimensional variation of resistivity with depth (sounding) starts with the potential on the surface:

$$V(r) = \frac{pI}{2\pi r} + 2 \int_0^{\infty} K(L)J_0(Lr)dL \quad (1)$$

where  $V(r)$  is the potential at distance  $r$  from a source to a measurement,  $p$  is the surface resistivity,  $I$  is current,  $L$  is a dummy variable of integration,  $J_0$  is a Bessel function (order 0) and  $K(L)$  is a characteristic function of the thickness and resistivity of the subsurface layers. Rewriting equation (1) to give the apparent resistivity for a Schlumberger array (O'Neill, 1975) yields:

$$p_a(s) = s^2 \int_0^{\infty} T(L)J_1(Ls)LdL. \quad (2)$$

The variable  $T(L) = p(1 + 2K(L))$  is easily calculated for an arbitrary number of layers as shown below,  $J_1$  is a Bessel function (order 1), and  $s$  is half the separation of the current electrodes. This integral is time consuming to evaluate because the Bessel function in the integrand oscillates and converges very slowly for many geologically reasonable models. The forward problem used to be solved by evaluating this integral numerically. The only method in use today involves the substitutions  $x = \ln(s)$  and  $z = \ln(1/L)$  to convert it to a convolution integral such as:

$$p_a(x) = \int_{-\infty}^{\infty} T(z) J_0(e^{x-z}) e^{2(x-z)} dz. \quad (3)$$

This is a standard form for linear filtering. The apparent resistivities are the output, the resistivity transform from the model is the input, the remaining term in the integrand is the filter function. O'Neill (1975) provides 20 coefficients (Table 1) for the filter function for a Schlumberger array. Writing equation (3) in an algebraic form suitable for programming or spreadsheet use yields:

$$p_a(m) = \sum_{i=-5}^{14} b_i T_{m-i}. \quad (4)$$

Here, the  $b_i$ 's are the filter coefficients. For each calculated value  $m$ , the resistivity transform  $T_m$  must be calculated for enough samples to accommodate 14 leading and 5 trailing filter coefficients.

The resistivity transform for a model can be calculated a number of different ways. A convenient form (Das and Ghosh, 1974) is:

$$T_{i-1} = \frac{T_i + p_{i-1} \tanh(Ld_{i-1})}{1 + \tanh(Ld_{i-1})/p_{i-1}}.$$

One starts with the bottom layer in this recursive relation and proceeds layer by layer to the top. The dimension of  $L$  is inverse distance,  $p_i$  and  $d_i$  are the resistivity and thickness of the layer, respectively. For a three layer case one would start with  $i = 3$ , and thus  $T_i = p_3$  which is the resistivity of the bottom layer. Next one calculates the transform for the second layer ( $T_2$ ), and finally the top layer ( $T_1$ ):

$$T_1 = \frac{T_2 + P_1 \tanh(Ld_1)}{1 + T_2 \tanh(Ld_1)/p_1}. \quad (6)$$

$$T_2 = \frac{p_3 + p_2 \tanh(Ld_2)}{1 + p_3 \tanh(Ld_2)/p_2}$$

To use O'Neill's (1975) Schlumberger filter coefficients, the resistivity transform is calculated for six evenly spaced samples per integer power of ten. Thus,  $T_1$  is calculated six times (6 values of L) for each order of magnitude of electrode spacing.

### **PROGRAMMING THE SPREADSHEET**

Figure 1, generated with Bordland's Quattro Pro™ spreadsheet running on an IBM/MS-DOS™ compatible computer, illustrates one setup for computing the convolution with a spreadsheet. The technique is practical on any computer with a spreadsheet. The necessary formulas for calculating the apparent resistivity are shown at the bottom of Figure 1. Most spreadsheet features are intuitive and easily learned. Each cell has an address (column and row) and may contain a formula, constant, or label. Formulas in a cell can refer to any other cell. One way spreadsheets differ from other programming languages is by using relative and absolute cell addresses. When cell addresses are preceded by a \$ (e.g., \$B\$3) these are absolute row and/or column references. When copied to another cell in the spreadsheet, absolute row and column references do not change. Without the \$'s, the addresses are relative to position and the specified row and/or column references change when copied.

To calculate the expected apparent resistivity from a three-layer model, place the layer parameters in a convenient spot near the top of the spreadsheet. These model parameters are the input to the calculation; change these values to try a new model. The model parameters, the filter coefficients, the seed, and the increment are always used with absolute addresses.

Beneath the model parameters (Figure 1) are two constants. The seed accommodates the phase shift of O'Neill's (1975) filter coefficients. Input values (L's) at position x are associated with

output values (apparent resistivity) at  $x + \delta x$ . Using 2.556757 for the seed will put the first apparent resistivity value at a Schlumberger electrode half-spacing of one meter. Xinc (-.166666) gives the requisite six samples per integer power of ten.

The labelled columns in Figure 1 step through the calculation for the apparent resistivity; they progress from left to right. Figure 1 includes some example formulas for these columns. The first column is just the row number. The 38 rows provide apparent resistivities for electrode half-spacings from 1 to 1000 meters. The second column, L, uses the seed, row, and Xinc (use absolute references) to calculate L's for the resistivity transform. The third column (cryptically labelled: Ld2>230?) tests the product of L and the thickness of layer 2 so it does not get out of range for the exponentiation operator (@EXP). The next two columns calculate a hyperbolic tangent and the first step (T2) in the resistivity transform (equation 6), respectively. Next, two familiar columns yield the resistivity transform (T1) at the surface. One only has to type in the formulas (bottom of Figure 1) once; the COPY command will map them to the other 37 rows.

The convolution formula (Figure 1) assumes the 38 values of the resistivity transform are in a column next to the filter coefficients (Table 1). However, because the filter coefficients have absolute addresses, they can be in any convenient spot. Envision the filter coefficients as a one-dimensional array (or column) next to the column of values containing the resistivity transform. The convolution is the sum of products for each filter coefficient with the adjacent value of the resistivity transform. The output value (apparent resistivity) is fixed next to O'Neill's coefficient  $b_0$ . For each output value, the array of filter coefficients advances one step ahead of its previous position. The first apparent resistivity, for a Schlumberger half-spacing of one meter, is calculated for the row of transform values where  $n = 15$  and  $L = 1.1396$ . The last value, for a Schlumberger

**Table 1. O'Neill's (1975) filter coefficients. Their sum should be unity.**

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b <sub>-5</sub>	0.003042	b <sub>5</sub>	2.7044
b <sub>-4</sub>	-0.001198	b <sub>6</sub>	-1.1324
b <sub>-3</sub>	0.01284	b <sub>7</sub>	0.3930
b <sub>-2</sub>	0.0235	b <sub>8</sub>	-0.1436
b <sub>-1</sub>	0.08688	b <sub>9</sub>	0.05812
b <sub>0</sub>	0.2374	b <sub>10</sub>	-0.02521
b <sub>1</sub>	0.6194	b <sub>11</sub>	0.01125
b <sub>2</sub>	1.1817	b <sub>12</sub>	-0.004978
b <sub>3</sub>	0.4248	b <sub>13</sub>	0.002072
b <sub>4</sub>	-3.4507	b <sub>14</sub>	-0.000318

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half-spacing of 1000 meters, is at  $n = 33$  ( $L = 1.1396 \times 10^{-3}$ ).

The final formula in the example calculates the electrode spacing against which to plot the apparent resistivities. This last formula is similar to the formula for  $L$  except that part of the seed due to the phase shift (0.056758) is omitted. One should position the measured apparent resistivities, from experiments in the field, in a column and plot them with the model's apparent resistivities (Figure 2). If you want to extend the capabilities of the spreadsheet, filter coefficients for the Wenner electrode array are available in Ghosh (1971b); for dipole-dipole sounding coefficients see Das and Ghosh (1974).

Once the spreadsheet is operating properly, set up a graph of electrode spacing versus apparent resistivity and include a column for the data. When satisfied with the graph, imbed or insert it in the spreadsheet (Figure 2). Every time you alter one of the model parameters you get instant graphics-gratification. On a 20 mhz 386-387 personal computer, the recalculation and graphing takes less than one second. With one-second turnaround, it does not take long to match field

observations. This setup is also an excellent tool for designing sounding experiments. Playing "what-if" allows you to determine if sounding will be useful for a specific problem.

The real appeal of programming the problem on a spreadsheet is its simplicity and ease of programming. For comparison, a commercial forward and inverse sounding program like SOUNDER™ has about 30 lines of code to calculate the resistivity transform and convolution. SOUNDER™ has over 1000 more lines for the graphics interface; the forward calculation with graphics can be set up on the spreadsheet in about 20 minutes. By using absolute and relative addresses, the built-in math functions, and the logic operators, a commercial spreadsheet is a convenient and rapid high-level programming language for a wide variety of scientific problems. The immediate availability of a well designed graphics screen removes a tedious and time-consuming step from many geologic programming problems.

	B	C	D	E	F	G	H	I	J	K	L	M	N	P
2														
3				Three layer resistivity example. Depth and distance are										
4				in meters, resistivity is in ohm-meters. Apparent										
5				resistivity is for the Schlumberger electrode arrangement.										
6				Layer resistivities and thickness, from top to bottom:										
7														
8				Rho1 =	100	d1 =	10	(thickness of layer 1)						
9				Rho2 =	10	d2 =	20	(thickness of layer 2)						
10				Rho3 =	1000	ohm-meters								
11														
12				seed:	2.556757 includes phase shift for O'Neill's coefficients									
13				Xinc:	-0.1666667 to get 6 samples per integer power of ten									
14														
15		n	L	Ld2>230?	tanh(Ld2)	T2	Ld1>230?	tanh(Ld1)	T1	b	O'Neill	convolve	x=1/L	
16		1	2.5E+02	2.3E+02	1.0E+00	1.0E+01	2.3E+02	1.0E+00	100	14	-0.000318			
17		2	1.7E+02	2.3E+02	1.0E+00	1.0E+01	2.3E+02	1.0E+00	100	13	0.002072			
18		rows 18 through 28 omitted from figure												
19		14	1.7E+00	3.3E+01	1.0E+00	1.0E+01	1.7E+01	1.0E+00	100	1	0.6194			
20		15	1.1E+00	2.3E+01	1.0E+00	1.0E+01	1.1E+01	1.0E+00	100	0	0.2374	100	1	
21		16	7.8E-01	1.6E+01	1.0E+00	1.0E+01	7.8E+00	1.0E+00	100	-1	0.08688	100	1	
22		rows 32 through 53 omitted from figure												
23		example formulas from row 30 (n = 15):												
24		L	$10^{(2.5 + C30 * E13)}$											
25		Ld2>230?	$IF(D30 * G9 > 230, 230, D30 * G9)$											
26		tanh(Ld2?)	$(EXP(-E30) - EXP(E30)) / (EXP(E30) + EXP(-E30))$											
27		T2	$(E10 + E9 * F30) / (1 + (E10 * F30) / E9)$											
28		Ld1>230?	$IF(D30 * G8 > 230, 230, D30 * G8)$											
29		tanh(Ld1?)	$(EXP(-H30) - EXP(H30)) / (EXP(H30) + EXP(-H30))$											
30		T1	$(G30 + E8 * I30) / (1 + (G30 * I30) / E8)$											
31		b, O'Neill	(O'Neill's coefficient number and the coefficient)											
32		Convolve	$+J16 * L16 + J17 * L17 + J18 * L18 + \dots + J34 * L34 + J35 * L35$											
33		x = 1/L	$1 / (10^{(2.5 + C30 * E13)})$											

Figure 1

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
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## FIGURE CAPTIONS

**Fig. 1. Partial spreadsheet set up to calculate the apparent resistivity over a 3-layer model. The formulas at the bottom are extracted from columns with similar labels. The next (row = 31, n = 16) equation under convolve would use  $J17*\$L\$16 + J18*\$L\$17 + \dots$ . See the text for further discussion of symbols and equations.**

**Fig. 2. Completed spreadsheet set up for interactive modeling. Rho's are the resistivities assigned to the layers; d1 and d2 are their thicknesses. Electrode spacing is one-half the separation of the current electrodes.**



*Steven D. Sheriff is an Associate Professor of Geology at the University of Montana. He received a B.A. degree from Central Washington University in 1973, an M.S. degree from Western Washington University in 1976, and a Ph.D. from the University of Wyoming in 1981. His research interests include magnetic, gravity and electrical interpretation and modeling from the ground-water through crustal scale.*