Doctoral Dissertation Defense

"Spectral Preserver Problems in Uniform Algebras"

by

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There has been much interest in characterizing maps between Banach algebras that preserve a certain equation or family of elements. There is a rich history in such problems that assume the map to be linear, so called linear preserver problems. More recently, there has been an interest in not assuming the map is linear a priori and instead to assume it preserves some equation involving the spectrum, a portion of the spectrum, or the norm.

After a brief introduction to uniform algebras, we give a rigorous development of the theory of boundaries. This includes a new alternative proof of the famous Shilov Theorem. Also a generalization of Bishop's Lemma is given and proved. Two spectral preserver problems are introduced and solved for the class of uniform algebras. One of these problems is given in terms of a portion of the spectrum called the peripheral spectrum. The other is given by a norm condition.

The first spectral preserver problem concerns weakly-peripherally multiplicative maps between uniform algebras. These are maps $T : A \rightarrow B$ such that $\sigma_\pi(TfTg) \cap \sigma_\pi(fg) \neq \emptyset$ for all $f, g \in A$ where $\sigma_\pi(f)$ is the peripherial spectrum of *f*. It is proven that if *T* is a weakly-peripherally multiplicative map (not necessarily linear) that preserves the family of peak functions then it is an isometric algebra isomorphism.

The second of these preserver problems shows that if $T : A \rightarrow B$ is a map (not necessarily linear) between uniform algebras such that $\|TfTg + 1\| = \|fg + 1\|$ for all $f, g \in A$ then *T* is a weighted composition operator composed with a conjugation operator. In particular, if $T(1) = 1$ and $T(i) = i$ then *T* also is an isometric algebra isomorphism.

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3:10 – 5:00 pm in Math 103