The University of Montana

MATHEMATICAL SCIENCES

Presentation of Master's Thesis

Topos-like Properties in Two Categories of Graphs and Graph-like Features in an Abstract Category

By

Demitri Plessas

Often in the study of graph theory, the usual notion of a graph is that of a simple graph with at most one edge between vertices, and at most one loop on any vertex (some say no loops). The usual notion of a graph homomorphism is a mapping of graphs that sends vertices to vertices, edges to edges, and preserves incidence of the mapped vertices and edges. A more general view is to create a category of objects and morphisms that allows the graphs (i.e. the objects) to have multiple edges between two vertices and multiple loops at a vertex, coupled with more general graph homomorphism (i.e. the morphisms) that allows edges to be mapped to vertices as long as that map still preserves incidence, and of course, still maps vertices to vertices. Here this more general category of graphs is named the Category of Conceptual Graphs.

We investigate three topos defining properties of two subcategories of the Category of Conceptual Graphs. The first subcategory is the Category of Simple Loopless Graphs with Strict Morphisms in which the graphs are simple and loopless and the morphisms are restricted to only sending edges to edges (i.e. strictly), and the second subcategory is the Category of Simple Graphs with Strict Morphisms where at most one loop is allowed on a vertex. We find that these two graph categories have only a few topos-like properties. We also define some small graph-like objects in an abstract category that are their graph counterparts when viewed in any of the concrete categories of graphs. We also study these graph-like objects in some other familiar (concrete) categories, e.g. the Category of Abelian Groups and Homomorphisms and the Category of Sets and Functions.

May 5, 2008
3:10 pm in Math 108