

ACCEPTED Book Proposal

Shaping the Future of Mathematics Education

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This book will convey the state of the art on current theoretical perspectives in mathematics education and directions in which the future will/might shape up based on what is happening currently. The audience is the field of mathematics education [researchers, graduate and undergraduate students, as well as teachers].

Rationale

In recent years, the mathematics education research field has been criticized for its lack of focus, its diverging theoretical perspectives (and we would add, ineffective), and a continued identity crisis (Steen, 1999). Frank Lester has recently expressed grave concern over the lack of theoretical bases for many of the research studies reported in the literature (Lester, 2002, 2005). There are also distinct schools of thought with regard to mathematical thinking and learning that have evolved in Europe, in the U.S., and elsewhere. The theories proposed by these schools warrant closer attention with a view to extracting the most salient and timely aspects to generate directions for future developments in the field. At the dawn of this new millennium, the time seems ripe for our community to take stock of these major concerns and developments and chart significant courses for the future. In particular, we need to consider the important role of theories and theory building in research on mathematical thinking and learning within formal and informal environments.

Looking Back

Proponents of the period of Meaningful Learning (1930s and 1940s) advocated the development of mathematical learning with understanding, with William Brownell (e.g., 1945) emphasizing the importance of students appreciating and understanding the structure of mathematics. His recommendations are still highly relevant today: mathematics is the study of structure (Lesh & English, 2005). As highlighted in the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* (NCTM 2000), students need to learn mathematics with understanding by actively building new knowledge from existing knowledge and experience. The curriculum must be "more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades" (p. 14).

Van Engen's (e.g., 1949) advice, as well as that of the Gestaltists' (e.g., Wertheimer, 1959), is also highly pertinent to mathematics education today. For example, Van Engen emphasized the importance of developing students' ability to detect patterns in similar and seemingly diverse situations. In problem solving, students should identify the structure of a problem before searching for an answer. Likewise, the Gestaltists advocated the importance of productive thinking, as opposed to reproductive thinking, in the mathematics classroom (Wertheimer, 1959). A productive thinker grasps the structural relations in a problem or situation and then combines these parts into a dynamic whole. Such productive thinking can be encouraged by not giving students ready-made steps to solve given problems.

Developments during the period of the “New Mathematics” (1960s) still have significant input for mathematics education, despite the backlash they received in subsequent years. For example, Jerome Bruner’s (1960) recommendation for students to progress through three levels of representation, namely, the enactive, iconic, and symbolic, is still sound advice for effective curriculum development today. Likewise, the ideas of Zoltan Dienes (e.g., 1960), whose recent interview appeared in the journal, *Mathematical Thinking and Learning* (volume 9, issue 1), remain highly applicable. For example, Dienes placed a strong focus on multiple embodiments and on cyclic patterns of learning where students progress from concrete to symbolic formats in developing an understanding of mathematical structures.

Current State of Affairs

Developments in the Intervening Years, 1995-2007

There have been numerous developments in the intervening years that have either changed the landscape of mathematics education and/or provide significant pointers for future growth of our discipline. These developments include, among others: (a) a decreased emphasis on constructivism as the dominant paradigm for the teaching and learning of mathematics (e.g., Goldin, in press; Lesh & Doerr, 2003); (b) new developments in the learning sciences, in particular, a focus on complexity theory (e.g., English, in press; Jacobson & Wilensky, 2006; Lesh, 2006); (c) an increased focus on mathematical reasoning and interdisciplinary modeling (e.g., English, 2006; Lesh & English, 2005; Lesh & Zawojewski, 2007); (d) a significant increase in research on the mathematics needed in various work place settings and the implications for mathematics education (e.g., Gainsburg, 2006; Hoyles, Noss, & Pozzi, 2001; Hoyles, Bakker, Kent, & Noss, in press); (e) a broadening of theoretical perspectives, including an increased focus on social-cultural-political aspects of mathematics education (e.g., Greer, Verschaffel, & Mukhopadhyay; in press; Gutstein, 2007); (g) developments in research methodology, in particular, a focus on design research (e.g., Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Lesh & Sriraman, 2005; Shavelson, Phillips, Towne, & Feuer, 2003); and (h) the increased sophistication and availability of technology (e.g., Kaput, Hegedus & Lesh, in press; O’Neil & Perez, 2006).

Decreased emphasis on constructivism

The dominant influence of constructivist theories on mathematics education has waned in recent years. In their edited volume, *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*, Lesh and Doerr (2003) presented powerful arguments for why we need to move our field beyond the clutches of constructivist ideologies. For example, all of the goals of mathematics education do not need to be achieved through the processes of personal construction and not all the mathematics students learn need to be invented independently by students. In essence, construction is only one of many processes that contribute to the development of constructs.

Goldin (e.g., 2002, 2003, in press) has further highlighted the shortcomings of constructivist theories, explaining how radical constructivists:

rejected on *a priori* grounds all that is external to the “worlds of experience” of human individuals. Excluding the very possibility of knowledge about the real world, they dismissed unknowable, “objective reality” to focus instead on “experiential reality.”

Mathematical structures, as abstractions apart from individual knowers and problem solvers, were likewise to be rejected. In advocating the (wholly subjective) idea of “viability” they dismissed its counterpart, the notion of (objective) validity (Goldin, in press).

Paradigms, such as constructivism, which became fashionable in mathematics education over recent decades, tended to dismiss or deny the integrity of fundamental aspects of mathematical and scientific knowledge. I agree with Goldin that “It is time to abandon, knowledgeably and thoughtfully, the dismissive fads and fashions – the ‘isms’ – in favor of a unifying, non-ideological, scientific and eclectic approach to research, an approach that allows for the *consilience* of knowledge across the disciplines” (2003, p. 176).

A models and modeling perspective provides one such unifying approach to research in mathematics education. I address this shortly but first wish to mention briefly some significant developments in complexity theory that have important, as yet untouched, implications for mathematics education.

Developments in the learning sciences: A focus on complexity

In the past decade or so, the learning sciences have seen substantial growth in research on complex systems and complexity theories (e.g., Jacobson & Wilensky, 2006; Lesh, 2006). Such research is yet to have an impact on mathematics education. Clearly, we cannot ignore this body of research if we are to prepare our students effectively for their future lives. Our students live in a world that is increasingly governed by complex systems that are dynamic, self-organizing, and continually adapting. Financial corporations, political parties, education systems, and the World Wide Web are just a few examples of complex systems. In the 21st century, such systems are becoming increasingly important in the everyday lives of both children and adults. For all citizens, an appreciation and understanding of the world as interlocked complex systems is critical for making effective decisions about one’s life as both an individual and as a community member (Bar-Yam, 2004; Davis & Sumara, 2006; Jacobson & Wilensky, 2006; Lesh, 2006).

In basic terms, complexity is the study of systems of interconnected components whose behavior cannot be explained solely by the properties of their parts but from the behavior that arises from their interconnectedness; the field has led to significant scientific methodological advances (Sabelli, 2006). Educational leaders from different walks of life are emphasizing the need to develop students’ abilities to deal with complex systems for success beyond school. These abilities include: constructing, describing, explaining, manipulating, and predicting complex systems (such as sophisticated buying, leasing, and loan plans); working on multi-phase and multi-component projects in which planning, monitoring, and communicating are critical for success; and adapting rapidly to ever-evolving conceptual tools (or complex artifacts) and resources (English, 2002; Gainsburg, 2006; Lesh & Doerr, 2003). One approach to developing such abilities is through mathematical modelling, which is central to the study of complexity and to modern science.

Looking Forward

Increased focus on mathematical reasoning and interdisciplinary modeling

Modelling is increasingly recognized as a powerful vehicle for not only promoting students' understanding of a wide range of key mathematical and scientific constructs, but also for helping them appreciate the potential of mathematics as a critical tool for analyzing important issues in their lives, communities, and society in general (Greer, Verschaffel, & Mukhopadhyay, in press; Romberg et al., 2005).

The terms, *models* and *modeling*, have been used variously in the literature, including with reference to solving word problems, conducting mathematical simulations, creating representations of problem situations, and creating internal, psychological representations while solving a particular problem (e.g., Doerr & Tripp, 1999; Gravemeijer, 1999; Greer, 1997; Lesh & Doerr, 2003; Romberg et al., 2005). In my research in recent years I have defined models as "systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behavior of some other familiar system" (Doerr & English, 2003, p.112). From this perspective, modeling problems are realistically complex situations where the problem solver engages in mathematical thinking beyond the usual school experience and where the products to be generated often include complex artifacts or conceptual tools that are needed for some purpose, or to accomplish some goal (Lesh & Zawojewski, 2007).

Students' development of powerful models should be regarded as among the most significant goals of mathematics education (Lesh & Sriraman, 2005). Importantly, modeling needs to be integrated within the *elementary* school curriculum and not reserved for the secondary school years and beyond as it has been traditionally. My recent research has shown that elementary school children are indeed capable of developing their own models and sense-making systems for dealing with complex problem situations (e.g., English, 2006; English & Watters, 2005). Mathematics education needs to give greater attention to developing the mathematical modeling abilities of younger children, especially given the increasing importance of modeling beyond the classroom.

Research on the mathematics needed in work place settings: Implications for mathematics education

Numerous researchers and employer groups have expressed concerns that educators are not giving adequate attention to the understandings and abilities that are needed for success beyond school. Research suggests that although professionals in mathematics-related fields draw upon their school learning, they do so in a flexible and creative manner, unlike the way in which they experienced mathematics in their school days (Gainsburg, 2006; Hall, 1999; Hamilton, in press; Noss, Hoyles, & Pozzi, 2002; Zawojewski & McCarthy, 2007). Furthermore, this research has indicated that such professionals draw upon interdisciplinary knowledge in solving problems and communicating their findings.

The advent of digital technologies is also changing the nature of the mathematics needed in the work place. These technological developments have led to both the addition of new mathematical competencies and the elimination of existing mathematical skills that were once part of the worker's toolkit (e.g., Jenkins, Clinton, Purushotma, & Weigel, 2006; Lombardi & Lombardi,

2007). Studies of the nature and role of mathematics used in the workplace and other everyday settings (e.g., nursing, engineering, grocery shopping, dieting, architecture, fish hatcheries) provide significant pointers for the future-orienting of mathematics education. In the final chapter of the second edition of the *Handbook of International Research in Mathematics Education* (English, in press), we list a number of powerful mathematical ideas for the 21st century that are indicated by these studies of mathematics in work place settings. Included in this list are: (a) working collaboratively on complex problems where planning, monitoring, and communicating are critical for success; (b) applying numerical and algebraic reasoning in an efficient, flexible, and creative manner; (c) generating, analysing, operating on, and transforming complex data sets; (d) applying an understanding of core ideas from ratio and proportion, probability, rate, change, accumulation, continuity, and limit; (e) constructing, describing, explaining, manipulating, and predicting complex systems; and (f) thinking critically and being able to make sound judgments.

Looking Forward

A broadening of theoretical perspectives: An increased focus on social-cultural-political aspects of mathematics education

In recent years we have seen a major shift within the field of mathematics education from a mainly psychological and pedagogical perspective towards one that encompasses the historical, cultural, social, and political contexts of both mathematics and mathematics education (e.g., English, in press; Greer, Verschaffel, & Mukhopadhyay; in press). This multitude of factors is having an unprecedented impact on mathematics education and its research endeavours. Many of our current educational problems continue to be fuelled by opposing values held by policy makers, program developers, professional groups, and community organizations (Greer et al., in press; Skovsmose & Valero, in press). When mathematics is intertwined with human contexts and practices, it follows that social accountability must be applied to the discipline (D'Ambrosio, 2007; Greer & Mukhopadhyay, 2003; Mukhopadhyay & Greer, 2007; Gutstein, 2007). One potentially rich opportunity to address this issue lies in the increased emphasis on the inclusion of real-world problems in school curricula that involve data handling, statistical reasoning, and mathematical modeling and applications. There are numerous real-world examples where students can use mathematics to analyze socially and culturally relevant problems (Greer & Mukhopadhyay (2003). For example, Mukhopadhyay and Greer (2007) have outlined how the issue of gun violence, in particular as it impacts on students, can be analyzed in relation to its socio-political contexts using mathematics as a critical tool.

In the final section of this preface I review briefly design research, which in recent years has had a powerful impact on studies aimed at improving the teaching and learning of mathematics.

Developments in research methodology: A focus on design research

In recent years, the field of mathematics education research has been viewed by several scholars as a design science akin to engineering and other emerging interdisciplinary fields (e.g., Cobb et al., 2003; Lesh & Sriraman, 2005; Hjalmarson & Lesh, in press). Design research typically involves creating opportunities for both “engineering” particular forms of learning and teaching and studying these forms systematically within the supportive contexts created (Cobb et al., 2003; Lesh & Clarke, 2000; Schorr & Koellner-Clarke, 2003). Such a process usually involves a series of “iterative design cycles,” in which trial outcomes are iteratively tested and revised in

progressing towards the improvement of mathematics teaching and learning (Lesh & Sriraman, 2005; Shavelson et al., 2003). Such developmental cycles leave auditable trails of documentation that reveal significant information about how and why the desired outcomes evolved.

The focus on design science has led to studies addressing the interaction of a variety of participants (e.g., students, teachers, researchers, curriculum developers), complex conceptual systems (e.g., complex programs of constructions, complex learning activities) and technology, all of which are influenced by certain social constraints and affordances (e.g., Schorr & Koellner-Clarke, 2003). Such studies contrast with previous research involving information-processing approaches that traced the cognitive growth of individuals in selected mathematical domains such as number (e.g., Simon & Klahr, 1995). In the intervening years, design science approaches have opened up a new world of mathematics education research—we now have a greater understanding of a “learning ecology” (Cobb et al., 2003). Mathematics education involves not just individual learners and teachers; rather, it involves complex, interacting systems of participants engaged in learning experiences of many types and at many levels of sophistication.

Design studies, however, have not been without their critics. Shavelson et al. (2003) warned that such studies, like all scientific research, must provide adequate warrants for the knowledge claims they make. As Shavelson et al. emphasized: “By their very nature, design studies are complex, multivariate, and interventionist, making warrants particularly difficult to establish” (p. 25). Furthermore, many of these studies rely on narrative accounts to relay and justify their findings; the veracity of such findings is not guaranteed. In addressing this concern, Shavelson et al., have provided a framework that links research questions evolving from design studies with corresponding research methods focused on validation of claims.

Included in their framework are considerations pertaining to: (a) “What is happening?” (e.g., characterizing a sample of students with a statistical sample, addressing the depth and breadth of a problem through survey, ethnographic, case study methods); (b) “Is there a systematic effect?” (addressing issues related to an intent to establish cause and effect); and (c) “Why or how is it happening?” (a question seeking a causal agent). In our efforts to move our discipline forward, we need to give serious consideration to concerns such as these, but at the same time, we need to ensure that design studies are not sidelined by policymakers’ request for stringent, scientifically controlled research. We cannot afford to lose the rich insights gained from design research studies in our discipline.

Increased Sophistication and Availability of Technology

Since the publication of the 1995 book, *Mathematics education: Models and Processes*, the increase in sophistication and availability of new technologies has been quite incredible. Such technological growth will escalate in years to come, making it difficult to predict what mathematics knowledge and understandings our students will need in even five years from now. What appears of increasing importance, however, is the need for students to solve a variety of unanticipated problems, to be innovative and adaptive in their dealings with the world, to communicate clearly their ideas and understandings in a variety of formats to a variety of audiences, and to understand and appreciate the viewpoints of others, both within the classroom context and globally.

Numerous opportunities are now available for both students and teachers to engage in mathematical experiences within international learning communities linked via videoconferencing and other computer networking facilities (e.g., see O’Neil & Perez, 2006). There are also increasing opportunities afforded by classroom connectivity where multiple device types enable numerous representations to be passed bi-directionally and flexibly among students and between students and the teacher within the classroom environment (Kaput, Hegedus, & Lesh, in press). Kaput et al. view classroom connectivity as “a critical means to unleash the long unrealized potential of computational media in education.”

As many researchers have emphasized, however, (e.g., Mauer, 2000; Moreno-Armella & Santos-Trigo, in press; Niss, 1999), the effective use of new technologies does not happen automatically and will not replace mathematics itself. Nor will technology lead to improvements in mathematical learning without improvements being made to the curriculum itself.

As students and teachers become more adept at capitalizing on technological opportunities, the more they need to understand, reflect on, and critically analyze their actions; and the more researchers need to address the impact of these technologies on students’ and teachers’ mathematical development (Niss, 1999).

Suggested chapters for inclusion in this book are as follows (titles are tentative). The chapters will be comprehensive to the extent possible to make the book an accessible introduction to the field of mathematics education and thus serve as both a resource book for researchers as well as a textbook for upper division and graduate courses.

Chapter 1. Why a book on shaping the future of mathematics education?

Chapter 2 Crossroads in current theories of mathematical thinking and learning.

Chapter 3. The critical role of theory and philosophy in conceptualizing and conducting research.

Chapter 4. The impact of cultural, social, and political forces on theory development in mathematical thinking and learning.

Chapter 5. Neuroscience and mathematical thinking and learning.

Chapter 6. Design science paradigms for learning and teaching mathematics from a European perspective.

Chapter 7. Design science paradigms for learning and teaching mathematics (from a U.S perspective).

Chapter 8 Comparative perspectives on mathematical modeling. This chapter will address the different perspectives on math modeling from Europe and the US.

Chapter 9. Mathematical modeling and its application to the solution of cross-disciplinary problems.

Chapter 10. Concluding chapter that summarizes the key ideas presented in the book and recommends directions for future development.

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