Tracing students' Modelling processes in elementary and secondary school

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This study examines 6th and 8th grade students' mathematization processes as they worked a mathematical modelling problem. Students participated in four 40 minute sessions for deciding on the best city to live in among a number of different cities. In the present study we report on an analysis of the mathematization processes and developments of two groups of students, one 6th and one 8th grade, as they worked the problem, with special emphasis on the similarities and differences between the two groups. Results provide evidence that students developed the necessary mathematical constructs and processes to actively engage and solve the problem through meaningful problem solving. Both groups of students set hypotheses, evaluate, modify and refine their models. Among the differences between the two groups, 8th grade students were involved in higher level of mathematical communication, projected and effectively employed higher order mathematical concepts and processes and reached better and more refined solutions.

INTRODUCTION

In the present study we aim to study how students in elementary and secondary school work on modeling problems. According to our knowledge, so far only a limited number of research studies focused on students' developments through their work on modeling problems in elementary school (English, 2006; Doerr & English, 2003). Findings from these studies indicated that students in elementary school can effectively work in modeling problems and therefore, researchers stressed that modeling problems should be included in elementary school's mathematics. The present study aims to build on these prior findings, by tracing similarities and differences between two groups of elementary and secondary school students, while working on a modeling problem. The identification of these similarities and differences is expected to further contribute to the appropriate introduction of modeling problems in elementary and secondary school mathematics.

THEORETICAL FRAMEWORK

An increasing number of mathematics education researchers have begun focusing their research efforts on mathematical modelling, especially at the school level. This is evident in numerous research publications from groups of researchers in Australia (English, Galbraith and colleagues), Belgium (Verschaeffel and colleagues), Denmark (Niss, Blomhøj and colleagues), Germany (Blum, Kaiser and colleagues), Netherlands (de Lange and colleagues) and the U.S. (Lesh, Schoenfeld and colleagues), among many others.

A very promising idea, coming from Blum and Niss (1991) documented the importance of modeling as a problem solving activity. Blum and Niss (1991) reported that there is a strong need to implement worthwhile modelling experiences in the elementary and middle school years if teachers are to make mathematical modelling a successful way of problem solving for students. Recent research indicated that student work with modeling activities assisted students to build on their existing understandings, and to develop important mathematical ideas and processes that students normally would not meet in the traditional school curriculum (Zawojewski, Lesh, & English, 2003; Lesh & Sriraman, 2006). As students work in these activities, they engage in important mathematical processes such as describing, analyzing, coordinating, explaining, constructing, and reasoning critically as they mathematize objects, relations and patterns (Mousoulides, Pittalis & Christou, 2006).

A number of researchers stressed the appropriateness of modeling activities for elementary and middle school students (English, 2006; Doerr & English, 2006). English and Watters (2005) reported that there was considerable evidence, in their research with young learners, that students' mathematical ideas had improved after they worked in a sequence of modeling activities. Since students' work in modeling activities is not narrowed only in working with ready made models, students need to construct models in a meaningful way for solving a real problem. This construction can lead to conceptual understanding and mathematization (Lesh & Doerr, 2003; Lesh & Sriraman, 2006). Doerr and English (2003) reported that modeling activities provide opportunities for elementary school students to explore quantitative relationships, analyze change, and identify, describe, and compare varying rates of change, as recommended in the Grades 3-5 algebra strand of the Principles and Standards for School Mathematics (NCTM, 2000). In addition, English (2003) pointed that elementary probability ideas emerging when young students linked the conditions and constrains of problems.

Another important parameter is students' use of their informal knowledge in solving modeling problems. Mousoulides and colleagues (2006) reported that students' informal knowledge helped them relate to and identify the important problem information (e.g., understanding and interpreting the conditions for the solution of a problem). The interplay helped students in finding solutions for the assigned problems and refining solutions accordingly to meet the necessary real world restrictions and criteria (Doerr & English, 2003; Zawojewski, et al., 2003).

As a concluding point, modeling activities provide a pathway in understanding how students approach a mathematical task and how their ideas develop; these activities appear to provide a strong basis for teachers to interact with students in ways that would promote their learning (Kaiser & Sriraman, 2006; Doerr & English, 2006). The latter is among the core aims of mathematics education.

THE PRESENT STUDY

The Purpose of the Study

The aim of the present study is to explore the similarities and differences between elementary and secondary school students, while they work on an authentic modeling problem. To this end, it is expected from both groups of students to work with authentic mathematical problems, using their prior mathematical knowledge to investigate, make sense and understand these problems. In other words, in the focus of the present study is the tracing of the aforementioned similarities and differences in an attempt to explain why these similarities and differences might appear and to explore possible reasons for that. The results of the study are expected to contribute to current research in the area of introducing modeling as an appropriate and successful approach in mathematical problem solving for elementary and secondary school students.

Participants and Modeling Activities

Thirty seven students (22 females and 15 males) from two intact 6th and 8th grade classes in two urban schools in Cyprus participated in one modeling activity, presented below. All students had little experience in solving problems in a mathematical modeling context, since both classes are participating in a larger project on the effectiveness of mathematical modeling in problem solving.

For the purposes of this study, student work on one modeling activity will be presented, namely the "The Best City" activity. The activity is a modified version of one activity derived from a list found in Lesh and Doerr (2003). The purpose of the activity was to provide opportunities for students to organize and explore data, to use statistical reasoning and to develop appropriate models for solving the problem. Additionally, the activity provided a setting for students to focus and work with the notions of ranking, selecting, aggregating ranked quantities and weighting ranks.

The application of the "The Best City" activity (see Figure 1) followed three stages: (a) the warm-up stage in which students read an article with the purpose to familiarize themselves with the context of the modeling activity and to answer readiness questions through a whole class discussion, (b) the modeling stage in which students were engaged in constructing models to solve the activity, and (c) the presentation and discussion stage in which students presented their solutions and reflect on other student solutions.

Procedure

The students spent around 160 minutes (four 40 minute sessions) in completing the modeling activity. The activity started with a whole class discussion on the warm-up task and readiness questions on the related article (this stage lasted around 20 minutes for both groups of students). The second part of the modeling activity was the modeling stage. During this stage, which lasted around 80-90 minutes (for both groups), students worked in groups of three or four to provide solutions for the activity. After completing their work, each group presented its solutions to the rest of the class for questioning, comparing with others' solutions and constructive feedback. Finally, a whole class discussion focused on the key mathematical ideas and processes that were developed during the modeling activity. This last stage of the activity, namely the presentation and discussion stage lasted around 45 minutes for the 6^{th} grade group and around 60 minutes for the 8^{th} grade group of students.

Data Sources and Analysis

The data for this study were collected through (a) videotapes of students' responses during whole class discussions, (b) audiotapes of students' work in their groups, (c) students' worksheets and final reports detailing the processes used in developing models and solutions, and (d) researchers' field notes. Videotapes and audiotapes were analyzed using interpretative techniques (Miles & Huberman, 1994), for evidence of students' mathematical developments towards the mathematical concepts appeared in the modeling activity. The analysis of the data was completed in several steps. First, all transcripts were reviewed by two researchers to identify the ways in which students interpreted and understood the problem, their approaches to selecting, categorizing, and aggregating the different factors, and their mathematization processes as they quantified factors, transformed factors, such as the next year's budget, and combined factors for creating "super factors" for kids and adults (see transcripts in results session). Second, all of the students written products in their worksheets were analyzed to identify and compare the mathematization processes used in their model development to obtain solutions to the problem and to compare solutions among the two groups (6^{th} and 8^{th} grade) of students.

Due to space limitations, we mainly present the results of one group of students in each grade, as they worked on the modeling stage of "The Best City" activity. Each group of students in each grade was selected on the basis of their provided solutions and their whole work. We have to report here that the selected groups were representative of the two classes, in a way that other groups in their classes reported similar work on the provided problem.

Use the data in the table below to find the best city, Anastasia can live in. When you reach an answer, write a letter, explaining and documenting your results, to Anastasia.

	Parks	Nursery Schools	Schools	Cinemas	Restaurants	Shops	Road quality (%)	Next year budget*
Lakecity	2	2	7	1	3	23	45.5	Same
Relaxcity	3	1	4	3	12	16	36.8	More
Safecity	2	4	5	4	4	26	57.2	Less
Dreamcity	0	5	10	0	6	12	19.7	Less
Nicecity	3	2	8	2	5	20	25.8	Less
Livecity	4	3	7	3	8	15	76.2	More

* Next year's budget is compared to this year's budget.

Figure 1: The Modeling Stage in "The Best City" activity

RESULTS

The results of the study are presented as follows: First, consideration is given to a "microlevel analysis" of the developments displayed by each group of students in working with the modeling activity. Following, this fine-grained microlevel analysis, a "macrolevel analysis" of the mathematization processes displayed by both groups of students is presented, to obtain possible similarities and differences between the different grade groups.

Microlevel analysis

Identifying and clarifying factors

Both groups commenced the question for finding the best place Anastasia could move on, by brainstorming on the factors presented in the table (see Figure 1), questioning the meaning and importance of these factors. In 6th grade group, a student pointed that parks and cinemas are important for a person: "Dreamcity neither has parks nor cinemas. I think that Dreamcity is the worst place for Anastasia". Similarly, students in 8th grade group reported that Dreamcity was the worst place for Anastasia, since: "No cinemas, no parks, few shops and bad roads". On the contrary, while 6th grade students did not discuss the meaning of increased or decreased next year budget, there was a long debate among 8th grade students to clarify what is the meaning of this factor, and most importantly, how this factor is related with other factors and most importantly how budget can influence other factors: “Look at Relaxcity. There are few parks but only one nursery school … this could change, since Relaxcity’s next year budget will increase […] People there can use these money to improve city’s facilities”.

A long discussion between the members of the 8th grade group questioned the representativeness of their ideas and solutions, related to the importance of certain factors. One girl pointer out that “having parks is it not important for me…having shops and cinemas is more important”. The same girl highlighted that Anastasia was a college graduate and therefore “many schools and nursery schools are not as important for her as shops, restaurants and cinemas”.

Beginning mathematization

After their first impressions, one student in the 6th grade group suggested that the group should focus on comparing two cities at each time. In doing so, students compared one factor every time to find out which city was “better” than the other one. “Dreamcity has more restaurants than Safecity. Its streets are, also, better than Safecity’s”. There were also attempts to compare more than two cities: “Livecity has more parks than all other cities and the road quality in Livecity is much better than road quality in other cities”.

On the other hand, students in 8th grade group presented more sophisticated ideas, such as “Adding horizontally the numbers for each city” and “finding the number of buildings and facilities in each city”. In doing so, students made simple calculations and compare their results: “Let’s sum the total number of buildings and facilities for each city. This is a way to find which city is the best one for Anastasia”. It has to be reported here that students attempted to take into consideration road quality and budget factors: “Safecity’s budget will be decreased next year and look at its roads. Quality is only 25%. Road quality can not improve, since they will not have more money to spend on it”.

Working with factors

As seen above, 8th grade students started mathematization earlier than 6th grade students. However, it was clear that both groups of students experienced difficulties in their efforts to work with factors such as next year’s budget and the quality of roads and to combine these factors with other factors. At a later stage, one 6th grade student reported in his worksheet: “We added the buildings in each city. Budget is an important factor. It means what they will do next year. We decided to keep this factor by itself, since we could not add it with buildings and roads”. Similarly, 6th grade students kept the road quality factor as it was, but in their discussions they referred to road quality by reporting that “Dreamcity has bad quality of roads and Livevity has good quality of roads”.

A number of differences appeared in 8th grade group’s discussion. As the group discussed the meaning of budget and tried to summarize the number of buildings in each city, their next attempts focused on trying to recode the road quality data. Students categorized road quality as “above average”, “average” and “below average”. This was helpful in transforming the different numbers into three categories and therefore in making use of this factor, in contrast with 6th grade students. More specifically, they added 15 points to the total number of buildings if city’s road average was above average (>60%), 10 points to average cities (from 40% to 60%) and 5 points to below average cities (<40%).

An interesting strategy was presented by 6th grade students after the first presentation of their solutions to the whole class. Instead of finding the total number of buildings for each city, they grouped factors as those being important for young people and those that are more important for adults. Therefore, factors like cinemas, restaurants and shops were categorized as “adult factors”.

Macrolevel analysis of mathematization processes

In this level of analysis, we primarily focus on the mathematization processes projected by both groups of students, during their work on the modeling activity.

Categorizing and Merging factors

As reported in more detail above, b oth groups of students categorized factors as either related to *buildings* (schools, restaurants), *facilities* (parks, roads) and *budget*. Of importance is the sub categories reported by the 6th grade group, who assigned labels as *“buildings/facilities for children and teenagers and for adults”*.

The approach presented by 6th grade students suggested that: “We need to find a way to merge all these columns (referring to the table). My idea is to sum the first 3 columns for each city and then the last 3 columns. If we use this method we will have 2 factors; the first will refer to children/teenagers and the second one to adults”. This idea was adopted and students were able to refine their prior solutions, since: “the second factor is more important for Anastasia; she is probably single and she does not have children”.

An interesting strategy was presented in 8th grade group’s discussion. They transformed, for example, the existing number of parks to numbers from 1-6, by assigning 1 to the city with the maximum number or parks, 2 to the second city and 6 to the city with the least number of parks. Accordingly, they assigned numbers from 1-6 for each factor, except for next year’s budget. Students continued, by adding these numbers to obtain a general factor. They clearly stated that: “the best city is the one with the *minimum* number”. When students were encouraged to also include next year’s budget, one student reported that: “Our solution would be much better, but this is not easy. How can we change this qualitative information (more, less, same) into numbers”? A second student added that it was not the case to add some points for cities with increased budget and subtract from cities with decreased budget, because “more” for one city’s budget is not necessarily the same like “more” for another city’s budget.

Aggregating and Ranking factors

In ranking the factors, 8th grade students first applied a point multiplication system, which was changed two times during student work in the modeling problem. The first system that was applied in an attempt to rank the different factors was to multiply each factor by a number from 1 to 8 (since they had to consider eight factors). The most important factor was multiplied by 8, the second most important by 7 etc. At a second attempt, students decided to “group” factors in terms of their importance. As a result, students grouped the eight factors in three groups and assigned a system similar to previous one, multiplying by 1, 2, and 3, considering the importance of each factor. More specifically, in this “cycle” of possible solutions, they multiplied by 3 the “building group”, by 2 the “facilities group” and by 1 the budget.

On the contrary, 6th grade students ranked the different factors only in a qualitative manner; they considered some factors being more important than others, but that distinction was done only in a qualitative way. For example, one student from the 6th grade group wrote: “Livecity is a better place than Dreamcity. Livecity might have fewer nursery schools, schools and shops than Dreamcity, but these things are not so important. Budget is important, Livecity’s budget will increase and Dreamcity’s will decrease”.

DISCUSSION AND CONCLUDING POINTS

There is a number of aspects of this study that have particular significance for the use of modeling in mathematical problem solving in elementary and secondary school mathematics. First, primary and lower secondary school students can successfully participate and satisfactorily solve mathematical modeling problems when presented as meaningful, real-world case studies. As presented earlier, the activity did not narrow students’ freedom and autonomy to approach and analyze the problem taking into account their prior and informal knowledge. Modeling problems, like the one used in this study, enable different trajectories of learning, with students’ mathematical understandings developing along multiple pathways. Students present different trajectories of learning and because modeling problems can be solved at different levels of sophistication, students can use a diversity of solution approaches; as a result, students of different achievement level can contribute to, and benefit from, the learning experiences these modeling activities offer.

A second aspect of the study is located in the similarities between the two groups of students as they worked in the modeling activity. Students’ work in both groups was impressive; they analyzed the problem using different viewing angles, set and test hypotheses, evaluate, modify and refine their models and solutions. Quite important was students’ engagement in self evaluation; both groups were constantly questioning the validity of their solutions, and wondering about the representativeness of their models.

The third significant aspect lies in the differences between 6th and 8th grade students in using and sharing their mathematical ideas and understandings. Although modeling problems are valuable because they provide a rich framework for developing and presenting students' mathematical skills, only 8th grade students explicitly presented and communicated a number of mathematical concepts and processes, and effectively applied them in solving the problem. The 8th grade group sufficiently used weighting and aggregating data, ranking factors and assigning scores in subgroups of factors. On the contrary, although 6th grade students presented implicitly a number of mathematical processes, they did not manage to effectively apply them in solving the problem, but they partially use them without much success.

Another difference between the work of the two groups of students was from the perspective of communication and assessment. Although in both groups, students adequately communicated their ideas and solutions, it was clear that in 8th grade group, students progressively assess and revise their current ways of thinking. As a result, by listening to and reflecting on their peers' suggestions and models, they undertake constructive assessment. The latter helped students to reach better and more refined solutions. It is not the case that 6th grade students did not communicate sufficiently, but this communication was mainly focused on subsets of information and on discussions on single factors, and therefore was not productive in terms of refining and improving student models.

In preparing students for being successful mathematical problem solvers, both for school mathematics as well as beyond school, teachers need to implement rich problem solving experiences starting from the elementary grades and continue to lower and higher elementary grades. Results from research work like this study that provide both teachers and curriculum designers with details on how students at different grade levels access higher order mathematical understandings and processes.

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