Course: M 118 Sec. 01 (CRN 7xxxx) 3 cr., Autumn 2013
Mathematics for Music Enthusiasts
TΘ 12:40–2:00pm in MATH 108

Instructor: Mark Kayll
Econtact: mark.kayll@umontana.edu

Office: MATH 209
406.243.2403

Hours: M 2:10–3:00pm, Th 2:10–3:00pm & by appointment
(tentative) (open for all course matters, including DSS accomm.)

Prerequisites: either: M 090 (MAT 005, Introductory Algebra) with a minimum B− grade;
or: M 095 (MAT 100, Intermediate Algebra); or: appropriate ALEKS score;
and: elementary music background, plus an open mind.


Material: Three separate units, based on: Numbers & Music; Mathematics & Music; Probability &
Music. We’ll use portions of the text as well as handouts.

Important Dates: Labor Day (no classes) Monday, 3 September;
last day to add by CyberBear w/o consent Wednesday, 5 September (5pm);
last day to add/drop by CyberBear, or select Audit grade option Monday, 17 September (5pm);
Election Day (no classes) Tuesday, 6 November;
Veterans’ Day (observed) Monday, 12 November;
Thanksgiving vacation 21–23 November;
last day to add/drop w/o dean’s sig Monday, 29 October (5pm);
Last class meeting (during finals) Monday, 10 December 3:20–5:20pm.

Description: Course topics will revolve around the interplay and connections between mathematics and
music. Here are some example questions to be considered: Why are there circles of fifths and fourths
but not thirds or tritones?; What is equal temperament?; How is the chromatic scale related to modular
arithmetic?; How is the musical staff like a logarithmic scale for pitch?; How are overtones related to the
integers?; How does harmony derive from the overtone series?; What are the mathematical relationships
between pitches in consonant intervals and chords?; What are the historical obstacles (going back to the
Greeks) to tuning a musical scale that gives a mathematically precise harmony in all keys?

If any of these questions intrigue you, and you are looking to satisfy your general education mathemat-
ical literacy requirement, then this might be the course for you.

Key musical and mathematical concepts will be introduced/reviewed as they are encountered.

Learning goals and outcomes: See final page of this syllabus.

Free tutoring: Math@Mansfield, Mansfield Library main floor
Monday–Thursday: 11:00am–3:00pm; Friday: 10:00am–12:00pm

(over)
Class attendance & activities: Attendance will be taken and will contribute to the “in-class work” portion of the grade. Class activities will include: discussion, group work, and lectures. Often group work will consist of worksheets which will also contribute to the “in-class work”. Participation is necessary; learning mathematics is similar to learning to play a musical instrument or a new sport: one learns by doing, not by watching.

Readings: Reading the text and handouts is essential for this course. Short pop quizzes based on the readings will be given, to encourage students to remain current.

Homework: Problems will be assigned based on the in-class work. Solution keys will be distributed after homework is due. Homework problems will be discussed regularly in class. Assignments will be collected and checked, but individual problems will not normally be graded. Keep in mind that the only way to learn mathematics is to do mathematics. This means that students should be prepared to spend some quality time thinking about mathematics.

I urge you from the outset to get into the habit of staying on schedule with your reading and homework. This will help you to maximize the material you’re able to absorb in class, meaning less effort in preparing for tests.

Assessment: Course grades will be based on homework assignments, in-class work, and three term tests. Traditional letter grades will be assigned using the +/− system (see UM catalog at www.umt.edu/catalog/acad/acadpolicy/default.html). UM’s policy on Incomplete grades will be followed (see UM catalog).

Sample grading schedule:

<table>
<thead>
<tr>
<th>Item</th>
<th>Date(s)</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homework</td>
<td>28 August — 7 December</td>
<td>20%</td>
</tr>
<tr>
<td>In-class work</td>
<td>28 August — 7 December</td>
<td>20%</td>
</tr>
<tr>
<td>Test # 1</td>
<td>late September</td>
<td>20%</td>
</tr>
<tr>
<td>Test # 2</td>
<td>late October</td>
<td>20%</td>
</tr>
<tr>
<td>Test # 3</td>
<td>finals week (early December)</td>
<td>20%</td>
</tr>
</tbody>
</table>

Accommodation: If you are a student with a disability who will require reasonable program modification in this course, please meet with Disability Services for Students in Lommasson 154 for assistance in developing a plan to address program modifications. If you are already working with Disability Services, please arrange to meet with me during my office hours to discuss reasonable modifications that may be necessary. For more information, visit the Disability Services website at life.umt.edu/dss.

General Remarks

On credit: If you’re taking this course as a general education requirement, you must choose ‘traditional letter grade’, not CR/NCR. A ‘D−’ grade is considered passing and will earn course credit, but it will not fulfill the Gen Ed requirement. A minimum grade of ‘C−’ is needed to fulfill the Gen Ed math literacy requirement.

On homework: You may work with others on homework problems, and you are encouraged to do so; however, Solutions should be written down privately in your own words.

On tests: Each test will be based on the material from one of the three units we will study.

On make-ups: Make-ups for tests will not be given unless there is a valid excuse cleared with the instructor prior to the test. At least one of your most detrimental homework/in-class work scores will be dropped; thus, there will be no make-ups for quizzes, homework, or class work.

On deadlines: Any stated deadlines will be firm; please do not ask for extensions.

On pets: Please do not bring pets to class.

On electronic devices: Cell phones must be turned off during class meetings and office hour visits. If you need a calculator during a test, a separate device from your cell phone must be used.

On conduct: All students need to be familiar with the Student Conduct Code; it can be found in the ‘A to Z Index’ on the UM home page. All students must practice academic honesty. Academic misconduct is subject to an academic penalty by the course instructor and/or a disciplinary sanction by the university.
Learning goals

1. To examine ways in which mathematics is used, to follow and understand logical arguments, and to solve problems and answer questions, especially concerned with music. This includes learning to formulate a problem precisely, to interpret solutions, and to make critical judgments in the face of competing formulations and solutions;
2. To explore and examine several areas of overlap between mathematics and music. This could include, but is not limited to, number theory and music, the mathematics of tuning and musical interval analysis, and the role of randomness and probability in musical composition;
3. To study concrete instances of ways in which mathematics has been used both historically and contemporarily in answering musical questions and in solving musical problems;
4. To increase both depth and breadth of musical knowledge and understanding by viewing the subject through a mathematical lens;
5. To attain a degree of mathematical literacy, including an ability to read mathematical material and write correctly using mathematical notation. To develop skills to reason and think mathematically in order to function effectively in our world.

Learning outcomes

1. Understand the elementary number theory governing the possibilities for musical circles (of fourths, fifths, etc.). Related mathematical concepts include divisibility, the Division Algorithm, prime factorization, greatest common divisors, least common multiples, the Euclidean Algorithm, modular (clock) arithmetic, solving linear congruences, and generating sets. The key related musical concept is that of a circle, such as the circle of fourths or the chromatic scale.
2. Learn the relationship between the overtone series in music and the set of positive integers; understand how this impacts the mathematical relationships between pitches in consonant intervals and chords.
3. Understand mathematics underlying musical tuning systems, particularly Pythagorean tuning and equal temperament (but not necessarily limited to these two systems). Gain facility in computing frequency ratios between musical intervals in different tuning systems; be able to compare and contrast tuning systems mathematically. Become familiar, mathematically, with the Pythagorean comma, and gain awareness of other musical commas.
4. Learn the mathematics behind the placement of frets on a fretted instrument. Related mathematical concepts include inverse proportionality and geometric series (finite and infinite). Apply this knowledge toward the solution of problems in luthier design.
5. Gain an elementary understanding of the mathematics governing the music compositional transformations of transposition ($T$), inversion ($I$), and regression (retrograde) ($R$). Become aware of the finite mathematical limit on the total number of distinct combinations of the musical operations $T$, $I$, and $R$.
6. Learn how to convert between frequency ratios (of musical intervals) and semitones or cents (as two common measures of musical differences). Related mathematical concepts include exponential and logarithm functions. Review the laws of exponentials and learn the three basic laws of logarithms. Apply these laws to solve equations involving exponentials and logarithms. Express the reason why musical intervals are additive in terms of the addition law of logarithms.
7. Understand the variety of ways that randomness has been used in musical composition, both in the eighteenth and twentieth centuries. Related probabilistic concepts include Bernoulli trials, Markov chains, and more exotic examples of unstructured randomness. Learn how to compute and reason about probabilities in these settings, especially as they relate to the musical contexts.
8. Criteria differentiating among different levels of understanding center on numerical scores on homework assignments, quizzes, and tests, as determined by the course instructor. These evaluation instruments are based on the learning goals and outcomes enumerated herein.