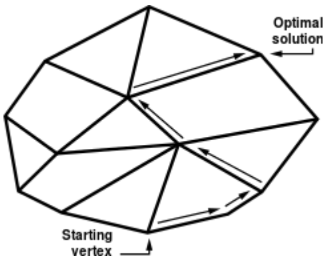
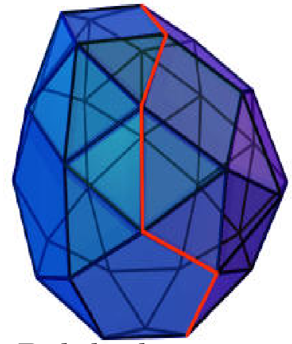
UNIVERSITY OF MONTANA

Department of Mathematical Sciences

Course Announcement – Spring 2024



Linear Inequalities & Convex Polyhedra



M 584 Section 01 Topics in C&O: Linear Inequalities and Convex Polyhedra
TΘ 11:00 am–12:20 pm in MATH 311
CRN 31581

Instructor: Mark Kayll `mark.kayll@umontana.edu` 406.243.2403

Why not start this announcement with a real $m \times n$ matrix A? Most math majors learn that the solution set to the equation $A\boldsymbol{x} = \boldsymbol{0}$ is a linear subspace of $\mathbb{R}^n$. What about the solution set to $A\boldsymbol{x} = \boldsymbol{b}$, when $\boldsymbol{b}$ is a *nonzero* element of $\mathbb{R}^m$? Or the solution sets to the systems of *inequalities* $A\boldsymbol{x} \leq \boldsymbol{0}$ and $A\boldsymbol{x} \leq \boldsymbol{b}$? Have you ever thought about these simple variations on the basic theme of "$A\boldsymbol{x} = \boldsymbol{0}$"? Maybe now is the time to take a good hard look at what can be gained by generalizing from elementary linear algebra.

Should you choose to enter this fascinating realm, your mind will be expanded by such concepts as 'affine subspaces', 'convex sets', and 'convex cones', by such techniques as 'Fourier-Motzkin elimination' and 'homogenization', and by famous theorems of Carathéodory, Radon, and Krein & Milman, not to mention a proof that the polytopes in $\mathbb{R}^n$ are precisely the bounded polyhedra (whatever that might mean!).

This subject features beautiful interplay between the geometry of higher-dimensional Euclidean spaces, the linear algebra needed for investigating these spaces, and the combinatorial aspects of the associated geometric objects. Though the discipline was born in the early 1900s, it remains an active research area, with, for example, the 1957 Hirsch Conjecture having finally fallen to *disproof* (by F. Santos) as recently as 2010.

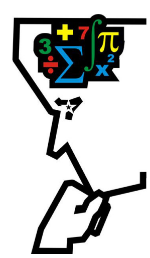
This course should be considered by anyone interested in the interplay between the subjects listed above or intrigued by this scintillating announcement.

Credits: 3

Prerequisites: interest and mathematical maturity
(at the level of a senior undergraduate or graduate student).

Grading: (few) homework assignments and a presentation (no exams).

Reference(s): mainly course notes, but several appropriate texts exist; e.g.,
A. Brøndsted, *An introduction to convex polytopes*, Springer-Verlag,
New York, 1983; ISBN-13 978-0387907222.



Questions? . . . Please! Just ask.