Course Announcement for Spring 2024:

M 522 — Advanced Algebra II

Time: MWF 1:00-1:50
Instructor: Nikolaus Vonessen

NOTE: This course is the continuation of M 521, but independent of it. If you have taken M 432, our second semester 400-level abstract algebra course, or a similar class, you are ready to take M 522.

M 522 is the second semester of the graduate introduction to abstract algebra. In the first semester, we covered groups, fields, and Galois Theory. This semester, we will study rings and modules, without assuming background from M 521.

In an earlier algebra class, you studied groups and subgroups. The role played by (normal) subgroups in group theory is played in ring theory by <u>ideals</u>. Ideals[1] are special examples of <u>modules</u>, a very important and powerful concept. In a nutshell, a module M over a ring R is like a vector space V over a field F. One can add and subtract the elements of a module (the "vectors"), and one can multiply them by scalars, but the scalars are allowed to belong to a ring R (say $R = \mathbb{Z}$, or $R = \mathbb{Z}_n$, the integers modulo n), they need not belong to a field (like $\mathbb{R}$ or $\mathbb{C}$). So unlike for vector spaces over fields, one usually cannot divide an element of a module by a non-zero scalar. We will spend quite a bit of time studying these objects.

Modules are quite useful. For example, given a finite-dimensional vector space V over a field F, and a linear operator T on V, we can give V the structure of a module over the polynomial ring $F[x]$, by letting x act via T. This allows us to study the linear operator T using the theory of modules over principal ideal domains, and to establish, for example, its Jordan Canonical Form.

TEXTBOOK: We will use the book *Introductory Lectures on Rings and Modules* by John A. Beachy, Cambridge University Press, 1999, ISBN-10: 0521644070, ISBN-13: 978-0521644075. This paperback currently sells online for about \$55 – \$75, plus shipping.

PREREQUISITES: This course assumes some familiarity with groups, rings, and vector spaces, but material from an undergraduate abstract algebra course (like M 432) will be reviewed as needed. Please contact me (`nikolaus.vonessen@umontana.edu` if you would like to take this course but are not sure if you have the necessary background.

[1]Yes, there is a reason for that funny name: The mathematician Kummer introduced them as "ideal numbers" which had better properties than the ordinary numbers (elements) in certain rings.