The University of Montana

MATHEMATICAL SCIENCES

Presentation of Master's Project

"Progress on the 123-Conjecture"

By

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A k-edge-weighting of a graph G is a map $w: E(G) \mapsto \{1,2,\ldots,k\}$. We say that a k-edge-weighting induces a proper coloring of $V(G)$ if for all adjacent vertices $u, v \in V(G)$ the sum of the weights of edges incident to u is different from the sum of the weights of edges incident to v. In 2004, Karoński, Łuczak, and Thomason conjectured that for any connected graph G such that $|V(G)| \geq 3$, there exists a 3-edge-weighting that induces a proper coloring of $V(G)$. This assertion is known as the 123-Conjecture. In 2010, Kalkowski, Karoński, and Pfender showed that for any such graph G there exists a 5-edge-weighting that induces a proper coloring of $V(G)$.

We confirm the 123-Conjecture for the Kneser graph, the generalized Kneser graph, and any complete k-partite graph. Our proofs make use of a technique of alternately weighting collections of edges with 1s and 3s. We also apply the Local Lemma to this problem to show that for 4-regular graphs, there exist 4-edge-weightings that induce proper colorings of the graphs. Additionally, addressing a question of Khatirinejad, Naserasr, Newman, Seamone, and Stevens, we show that for any tree T there are at least two non-isomorphic 3-edge-weightings that induce a proper coloring of $V(T)$.

Wednesday, May 7, 2014
3:10 pm in Math 108

Masters Committee

Dr. Cory Palmer, Chair (Mathematical Sciences),
Dr. Mark Kayll (Mathematical Sciences), Dr. George McRae (Mathematical Sciences)