UNIVERSITY OF MONTANA

MATHEMATICAL SCIENCES

Doctoral Dissertation Defense

"Digraphs and Homomorphisms: Cores, Colorings, and Constructions"

Michael Dennis Severino
Department of Mathematical Sciences

Wednesday, May 14, 2014
9:10 am in Math 312

A natural digraph analogue of the graph-theoretic concept of an 'independent set' is that of an acyclic set, namely a set of vertices not spanning a directed cycle. Hence a digraph analogue of a graph coloring is a decomposition of the vertex set into acyclic sets. In the spirit of a famous theorem of P. Erdős [Graph theory and probability, Canad. J. Math. 11:34–38, (1959)], it was shown probabilistically in [D. Bokal et al., The circular chromatic number of a digraph, J. Graph Theory **46**(3): 227–240, (2004)] that there exist digraphs with arbitrarily large digirth and chromatic number. Here we give a construction of such digraphs and define a new product of these highly chromatic digraphs with the directed analogue of the complete graph. This product gives a construction of uniquely n-colorable digraphs without short cycles.

The graph-theoretic notion of 'homomorphism' also gives rise to a digraph analogue. An acyclic homomorphism from a digraph D to a digraph H is a mapping $\varphi: V(D) \to V(H)$ such that $uv \in A(D)$ implies that either $\varphi(u)\varphi(v) \in A(H)$ or $\varphi(u) = \varphi(v)$, and all the 'fibers' $\varphi^{-1}(v)$, for $v \in V(H)$, of φ are acyclic. In this language, a core is a digraph D for which there does not exist an acyclic homomorphism from D to itself. Here we prove some basic results about digraph cores and construct new classes of cores. We also define a digraph-theoretic analogue to the graph-theoretic 'fractional chromatic number' and prove results relating it to other well-known digraph invariants. We see that it behaves similarly to the graph-theoretic analogue.

Dissertation Committee

Dr. Mark Kayll, Chair (Mathematical Sciences), Dr. Kelly McKinnie (Mathematical Sciences),
Dr. Jenny McNulty (Mathematical Sciences), Dr. George McRae (Mathematical Sciences),
Dr. Mike Rosulek (Computer Science – Oregon State University)