# 4 Montana Department of Mathematical Sciences 

## An Application of the Probabilistic Method of Proof: A graduate student describes his research project

## By Liam Rafferty



In May of 2005 I completed my B.A. in Mathematics at the University of Rochester. After finishing my undergraduate degree in a relatively large city, I decided that I didn't want to live in an urban environment anymore, so I moved to Missoula in 2005 to start my graduate studies in mathematics. I was surprised to find that Montanans consider Missoula a big city.

In 2007 I completed my M.A. studying the category of graphs under Professor McRae. Graphs are a combinatorial structure that can be used to model numerous counting problems. A graph is a set of vertices together with a set of edges, each edge incident on two vertices. The concept of a category is pretty abstract, and thus difficult to explain in a few words. Perhaps the most concise description of the category of graphs would be the collection of all graphs together with their structure preserving morphisms. My project was to examine the necessary conditions for an arbitrary category to be the category of sets, and see which of them were also present in the category of graphs.

After finishing my M.A., I shifted gears and, as an advisee of Professor Kayll, I began studying the application of the probabilistic method to a particular type of graph coloring problem. The probabilistic method is a type of nonconstructive existence proof.

What is a nonconstructive existence proof? Basically we
(Continued on page 6)


Jenny McNulty Wins MAA Meritorious Service Award

## By Nikolaus Vonessen



Professor Jenny McNulty, who also serves as an Associate Dean for UM's College or Arts and Sciences, was honored at the 2011 Joint Mathematics Meetings in New Orleans with a Certificate for Meritorious Service from the Pacific Northwest Section of the Mathematical Association of America (MAA). This award is given only once every six years.

Jenny has been active in the Pacific Northwest (PNW) Section for many years, including a term as Section chair from 1999 to 2001. During that time, she was instrumental in founding the PNW Section's Project NExT program. Project NExT (New Experiences in Teaching) is a professional development program of the MAA for new or recent PhDs
(Continued on page 3)

## Rick Billstein Wins Dennison Presidential Faculty Award

By Jim Hirstein


Rick Billstein received the 2011 George M. Dennison Presidential Faculty Award for Distinguished Accomplishment. Along with six other campus award recipients, Rick was honored at the UM Charter Day Ceremony in February. President Engstrom introduced Rick, and many of his friends and colleagues were there to congratulate him.

In his forty-two year association with The University of Montana, Rick has brought recognition to the university and the state of Montana. He has been a tireless ambassador for mathematics with over $\$ 9$ million in grants and countless talks and articles that promote his message. Rick directed the Six Through Eight Mathematics Project (STEM), a seven-year effort funded by the National Science
(Continued on page 3)

## Notes from the Chair

As usual, I would like to start these notes with congratulations to our graduates: 5 PhD students and 26 undergraduate math majors (including 2 students with a combined major in math and computer science) have finished their studies at the University of Montana this academic year. All our PhD recipients will have positions at academic institutions next year: Michael Gilliam is going to the College of New Rochelle in New York, and Nick Haverhals will be at Avila University in Kansas City, Missouri. At the time of this writing, Matt Roscoe had not yet decided which of his offers to accept. The remaining two, Demitri Plessas and Liam Rafferty, are staying with our department for next academic year: they applied for and were offered our two new (base budget funded) one-year postdoctoral lecturer positions.

Beginning in the fall, Bonnie Spence will be a lecturer in our department. Her specialty is Math Education; she has taught a variety of math courses in our department for a number of years. On behalf of the entire faculty I would like to welcome Bonnie and wish her all the best in this endeavor! Have a look at her interesting article about BuzzMath on the facing page.

I also would like to thank our generous donors for the gifts that we use to enhance students' educational and research experiences! The numerous awards, fellowships and travel grants
that we give to our graduate and undergraduate students would not be possible without your extraordinary support!

In the spring semester of 2011, for the first time in the history of our department (and of UM), a graduate course, Category Theory, was taught both face-to-face and online: the live lectures were videotaped, and the recordings were placed on the Mathematical Sciences website. (These lectures may be located by going to our department's web page and following the links: Classes with web pages; Spring 2011, Course M 595). Now everyone can watch these lectures online. This experiment will continue in the fall semester: three courses are going to be delivered in the same format; they will be videotaped and placed on the web, so that interested students can take these courses while being far away from campus.

The fall semester will be somewhat unusual for our department: no one will be gone either for exchange or for sabbatical; all faculty members are going to be on campus. This will give us an opportunity to reflect on our programs and activities with the goal of making our department stronger in research, instruction, and service.

I wish everyone a great summer!




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## BuzzMath, a Web-based Math Practice Site

## By Bonnie Spence



In the summer of 1992 I arrived at the University of Montana for the first time to write math curriculum for the STEM project under the guidance of Rick Billstein and Jim Williamson. This summer project led to a career in writing, and since then I have worked for the Math Department as a writer on an NSF grant project and also as an adjunct instructor. The experience I gained from the STEM project has led to many other writing opportunities outside the university. Two years ago I was approached by the Canadian company Scolab, Inc. to help them refine their French-Canadian product NetMaths and bring the product into the U.S. market as BuzzMath. Since then I have been writing and editing the curriculum, testing with local students, providing webinars, writing blogs, and presenting BuzzMath at national conferences.

BuzzMath is an online web-based math practice site that includes over 2500 high quality problems along with a math history theme that engages students through challenging "math missions". I was immediately impressed by the developers of the software and their vision for mathematics education. In viewing other middle school math software, I have noticed that many programs claim to raise test scores, but yet only present multiple choice responses and then reward students with games that are often unrelated to mathematics and detract from the real goal: to engage in mathematics. What I like about BuzzMath is that students interact and demonstrate their knowledge in a variety of
styles. They can manipulate circle graphs, interact with Cartesian coordinate systems and number lines, drag and drop integer tiles, and even rotate 3-D figures to count vertices and edges. Furthermore, BuzzMath randomizes the numerical values in their questions so that students on the same page are not all working the same problem. Students are allowed to study detailed feedback to a missed problem and then retry it with different values, which gives students an opportunity to demonstrate mastery. On top of all this there is a fun theme of unlocking missions that encourages more math engagement instead of distracting from the purpose of learning.

In January 2011, Google Apps Marketplace launched its new EDU education category with twenty featured apps, including BuzzMath. BuzzMath is offered on a subscription basis to schools and families, though it also has free accounts directly from its website http://www.BuzzMath.com that allow students to check up to 20 answers per hour.


## (Continued from page 1 "Rick Billstein")

Foundation. The project brought dozens of talented writers and evaluators to our campus. They wrote, edited, field tested, and revised an activity-driven, technology-enhanced, and assessment -enriched series of three textbooks and teaching ancillaries called Middle Grades MathThematics (1999), now in its second edition (2008).

Rick is also co-author of the book A Problem Solving Approach to Mathematics for Elementary School Teachers (with Libeskind and Lott). First published in 1981, this book quickly rose to the top in popularity for use in teacher training programs. It is currently in its tenth edition (2009) and is published in English and Spanish with teacher resources that include activities, tests, and technology packages. Here again, the thousands of teachers who learned their mathematics from Billstein, Libeskind and Lott have passed this knowledge on to untold millions of elementary school children for two decades, and will continue to do so for many years to come.

Rick's accomplishments reached a peak in the last eighteen years that would seem difficult to surpass. In the letters sup-
porting his nomination for this award, his career was described as distinguished by colleagues who are regionally and nationally recognized themselves. As he prepares for his retirement this year, winning the Dennison Presidential Faculty Award is a fitting tribute to Rick's dedication to mathematics education at The University of Montana.
(Continued from page 1 "Jenny McNulty")
in the mathematical sciences. Jenny has served as director of the PNW Section's Project NExT program since its inception in 2000. According to the prize citation, the "program continues to flourish and is a major reason that the Section has renewed vitality; revival of the Section can be marked with the April 2001 meeting, planned under Jenny's tenure as Section chair. As evidence of Jenny's continued impact on the Section, during the banquet at the 2010 Section meeting, attendees were asked to raise their hands if they were involved with Project NExT activities, and half the people in the room raised their hands. Jenny McNulty is the reason for all those hands."

Joseph Hashisaki Memorial Scholarship<br>Martin Schmidt<br>Anderson Mathematics Scholarship Briana Peck<br>Mac Johnson Family Scholarships<br>Soya Bjorlie<br>Brittany Elliot<br>Lauren Morey<br>N. J. Lennes Competition<br>$1^{\text {st }}$ Place (Tie)<br>Stephanie Bell Adam Clinch<br>$3^{d}$ Place (Tie)<br>Daniel Barthelmeh R. Amzi Jeffs<br>Undergraduate Teaching Scholars<br>Soya Bjorlie<br>Erika Blough<br>Lauren Morey<br>Martin Schmidt<br>Indergraduate Tutorial Scholars<br>Jesse Finneman<br>Kinnsey Reilly

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Rachel Robertson
Bertha Morton Scholarship
Marylesa Howard
John A. Peterson Awards for Mathematics Education
Elijah Bodish
Abram McCormick
Mortar Board - Outstanding Seniors

| Stephanie Bell | - | C \& O |
| :---: | :---: | :---: |
| Elijah Bodish | - | Math Ed |
| Adam Clinch | - | BA in Math |
| Sharee Russell | - | Applied Math |
| Andew Selle | - | Pi Mu Epsilon |
| Hannah Stanton | - | Pure Math |

## Degree Recipients 2010-2011

## BA in Mathematical Sciences

Tamatha R. Abell
Stephanie B. Bell
Kayleigh A. Blair
Elijah S. Bodish
Joshua P. Byrnes
Adam J. Clinch
Ted J. Clyde
Ryan W. Gramm
Vincent C. lannucci
Nicole F. Jefferson
Chase C. Maier
Jeremy C. Majerus

BS in Mathematical Sciences / Computer Science
Steven S. Garcia
Taylor B. Startin

Abram J. McCormick
Kelly P. McGonigal Travis N. Miller Lucy K. Muragin Windy R. Parnell
Deborah D. Perez

## PhD in Mathematical Sciences

The Szegö Kernel for Non-Pseudoconvex Domains in C²
Proof Trajectories-Results of a Longitudinal Qualitative Study
The Categories of Graphs
D-Colorable Digraphs with Large Girth
Informal Mathematics Activities and the Beliefs of Elementary Teacher Candidates

Adam E. Ruhnke Sharee N. Russell
Andrew C. Selle
Cory J. Smith
Hannah C. Stanton
Samantha B. Werme

Michael A. Gilliam
Nicolas J. Haverhals
Demitri J. Plessas
Liam T. Rafferty
Matthew B. Roscoe

## Alumni News

Maria H. Andersen (B.A. 1996) is the new "Teaching with Tech" columnist in MAA Focus, the newsmagazine of the Mathematical Association of America. She has also been appointed as the 2011 Professional Development Coordinator for AMATYC, the American Mathematical Association of Two-Year Colleges. She regularly shares her ideas and thoughts about teaching math on her blog, TeachingCollegeMath.com.

After graduating from UM, Michael Coons (B.A. 2003) pursued graduate studies in mathematics, earning an M.S. from Baylor University in Waco, Texas, in 2005. He then held a Fulbright Fellowship at the Alfred Renyi Institute of the Hungarian Academy of Sciences in Budapest, and in 2009 graduated with a Ph.D. in analytic number theory from Simon Fraser University in Burnaby, British Columbia. He currently holds one of the prestigious Fields-Ontario Postdoctoral Fellowships at the Fields Institute and the University of Waterloo in Ontario.

After teaching for one year at UM, John Hart (Ph.D. 2009) accepted a position as an assistant professor for two years at Sungkyunkwan University (SKKU) in Seoul, South Korea, the oldest university in East Asia. John's path to Korea was a bit convoluted: He had originally accepted a position at a university in Peru. But when he arrived there this January, he discovered that his contract had been unilaterally changed - his salary had been cut and his teaching load increased. So he said no thanks, and instead accepted his offer from SKKU.

We heard from Dave Perkins (Ph.D. 2005), who is a fulltime assistant professor of mathematics at Luzerne County Community College in Pennsylvania. The Mathematical Association of America just accepted his book Calculus \& Its Origins for publication in the Spectrum series congratulations!
Please send in your news; we're always glad to hear from you, and your classmates and professors would love to read about you in this column. N.V.

## Department News

Associate Professor John Bardsley is organizing a two-part mini-symposium Statistics, Computation, and Inverse Problems for the $7^{\text {th }}$ International Congress on Industrial and Applied Mathematics, which takes place in Vancouver, BC, in July 2011. Both he and his Ph.D. student Marylesa Howard will give talks in the mini-symposium. The international congress takes place every four years and is the world's largest applied/computational mathematics meeting.

Professor Bharath Sriraman was co-organizer of a 5day workshop "Teachers as Stakeholders in Mathematics Education Research" at the prestigious Banff International Research Station in Banff, Alberta. He continues as editor of the free online journal The Mathematics Enthusiast. The latest issue has an impressive list of well-known contributors, featuring, among others, an article by Hyman Bass, the author of the famous classic text Algebraic KTheory.

Professor Thomas Tonev gave an invited hour talk at the International Conference on Algebra, Topology and Topological Algebras held in January in Vera Cruz, Mexico.

Our new graduate program Master of Arts in Teaching Middle School Mathematics, which was described in the last newsletter, has received final approval from the Board of Regents. Three courses offered this summer will count towards this program.

This is the $10^{\text {th }}$ newsletter issue that I am editing, and will be the last (at least for a while): I will soon be stepping down from the position of Associate Chair for the Undergraduate Program which I have held for 5 years. I am grateful to Associate Professor Greg St. George for taking on this task. I enjoyed the job and the many challenges it brought, but am now ready for a change, and am looking forward to having once again more time for teaching and research.

Best regards,
Nikolaus Vonessen

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(Continued from page 1 - "An Application of the Probabilistic Method of Proof") prove the existence of something without actually finding it. Consider the following (trivial) conjecture: there exists a woman who voted for Max Baucus in the previous senatorial election. A constructive proof would be to go out and find such a voter. Unfortunately voting is an anonymous procedure, so we can't reliably prove that any particular woman voted for Baucus.

Perhaps a nonconstructive argument would be more convincing. Suppose we create a probability space out of the voters of Montana (any particular voter being chosen with equal likelihood). According to the vote counts the probability that a randomly chosen voter voted for Baucus is .73 . According to exit polling the probability that a randomly selected Montana voter in this particular election is a woman is .51 . Since both of these probabilities are greater than .5 , we conclude that there must be at least one voter who is both a woman and voted for Baucus.

Note that in the previous proof we never actually found such a person, we simply proved that such a person must exist. This is why we call it a nonconstructive method, we don't actually find what we're looking for, just prove that it exists.

Obviously I don't work at proving such trivial conjectures, but the basic idea is the same. Paul Erdős was an early developer of this method, and he used it to prove that there exist graphs with both arbitrarily large girth and arbitrarily large chromatic number.

The girth of a graph is the length of its shortest cycle and the chromatic number of a graph is the smallest number of colors necessary to color all the vertices such that no two adjacent vertices are the same color. For example,

the graph shown has girth 4 and chromatic number 3.
Intuitively it seems that these two properties would work against each other. In order to have large girth you wouldn't want there to be many edges; in order to get a large chromatic number you would want lots of edges. So it is surprising that there exist graphs with both properties.

My dissertation work is to generalize this theorem by Erdős to the setting of directed graphs. A directed graph (or digraph) is a graph where each edge has a direction assigned to it. Pictorially this is usually represented with an arrow on the edge. In our model the girth of a digraph is the length of a shortest directed cycle. We can generalize the concept of vertex coloring also; the acyclic chromatic number of a digraph is the smallest number of colors nec-
essary to color the vertices such that no directed cycle is monochromatic.

The generalized conjecture is then that there exist digraphs with both arbitrarily large girth and arbitrarily large acyclic chromatic number. It is not really necessary to understand what those properties are to understand the strategy we used to prove this conjecture. We constructed a probability space in such a way that each of those two properties existed with high probability, therefore there must exist a directed graph with both properties.

Liam Rafferty defended his Ph.D. thesis in May 2011. In the fall, he will join our faculty as a postdoctoral lecturer for one year.
(Continued from page 8 "Our Travels in Mathematics")
conferences we were able to present our research in the form of posters. These experiences were invaluable, from the students and professors we met to the tips we learned about employment and graduate school as well as the experience of standing in front of people and explaining our research.

Our research recently culminated in oral presentations at both the Center for Undergraduate Research in Mathematics (CURM) Research Conference held at Brigham Young University and the Pacific Coast Undergraduate Mathematics Conference (PCUMC) at Loyola Marymount University. The acquaintances we made at these conferences were again invaluable. Furthermore, the experience of giving a structured math talk within a 20 minute timeframe is a vital skill to develop as a future professional of any variety.

With all this talk of travel one might think, "Man I can't afford to do anything like this!" However, through multiple sources both on campus and off we have been able to fund our travel without much difficulty. The on campus sources consisted of the Montana Integrative Learning Experience for Students (MILES), the Provost Office, and our own math department. While the off campus sources consisted of funding from the conferences themselves, the Mathematical Association of America and the National Science Foundation. Most of these sources only required an application along with an abstract of one's research (which you'll need anyway at any conference!). Additionally to travel funding, CURM and MILES funds afforded us the opportunity to spend our time researching since portions of these grants were specifically allotted to compensate time spent researching.

We found research to be a great way to hone our analytical skills, strengthen our resumes, make our own contributions to mathematics, and gain experience beyond the classroom. We have thoroughly enjoyed and learned from the research process and all the opportunities it has allowed - Thanks to everyone for all of the support and encouragement!

## Alumni Reply Form

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The Department of Mathematical Sciences increasingly relies on donations to support its activities. With tuition increases continuing to outpace inflation, scholarships are vitally important. Scholarship and other existing funds that help to meet some of the educational needs of the department and our students are:

Joseph Hashisaki Fund - an endowed scholarship for one or two upper-class math majors based on academic achievement;
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Lennes Fund - an endowment to provide funds for the Lennes Exam;
Colloquium Fund - an endowment to provide funds to bring in visiting speakers.
Please consider a gift to one of these funds or to the Math Department's Excellence Fund to be used where the need is greatest.

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## Our Travels in Mathematics

## The Adventurers: Hannah Stanton, Stephanie Bell, Jordan Rooklyn \& Sharee Russell

Last year, we were invited by Professor Jenny McNulty to read a textbook she was working on in matroid theory. Our goal was to learn about and eventually do research in the field of matroid theory.

Matroid theory is a little known subject among undergraduates, but as we have learned, it is very accessible. Matroids are structures rooted in independence that combine several areas of mathematics. Anyone can take their favorite subject area - like graph theory, algebra, or combinatorics - and study matroids from that perspective (Wiki "Matroid" for more information, or even better, buy Jenny's textbook when it comes out!).

Over the course of the Spring ' 10 semester and the start of the Fall ' 10 semester we read Jenny's textbook and several other outside articles. Following this preliminary reading we were ready to begin our own undergraduate research.

We decided to examine various Maker-Breaker games and explore the structures of graphic and geometric matroids. From the middle of last fall through now, we have individually and collaboratively come up with unique conjectures and proven our own theorems.


From left to right: Sharee Russell, Jordan Rooklyn, Hanna Stanton, Professor Jenny McNulty and Stephanie Bell

We have had the opportunity to attend various research conferences across the U.S. One of those conferences was the Joint Mathematics Meetings (JMM) in New Orleans, LA. We also attended the Nebraska Conference for Undergraduate Women in Mathematics at the University of Nebraska-Lincoln where we were exposed to relevant topics to women mathematicians. At both of these
(Continued on page 6)

