Doctoral Dissertation Defense

"Peripherally-Multiplicative Spectral Preservers between Function Algebras"

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General sufficient conditions are established for maps between function algebras to be composition or weighted composition operators, which extend previous results regarding spectral conditions for maps between uniform algebras. Let X and Y be a locally compact Hausdorff spaces, where $A \subset C(X)$ and $B \subset C(Y)$ are function algebras, not necessarily with unit. Also let ∂A be the Shilov boundary of $A, \delta A$ the Choquet boundary of A, and $p(A)$ the set of p-points of A. A map $T : A \to B$ is called *weakly peripherally-multiplicative* if the peripheral spectra of fg and $TfTg$ has non-empty intersection for all f,g in A. (i.e. $\sigma_\pi(fg) \cap \sigma_\pi(TfTg) \neq \emptyset$ for all f,g in A) The map is said to be almost peripherally-multiplicative if the peripheral spectrum of fg is contained in the peripheral spectrum of $TfTg$ (or if the peripheral spectrum of $TfTg$ is contained in the peripheral spectrum of fg) for all f,g in A.

Let X be a locally compact Hausdorff space and $A \subset C(X)$ be a dense subalgebra of a function algebra, not necessarily with unit, such that $\delta A = p(A)$. We show that if $T : A \to B$ is a surjective map onto a function algebra $B \subset C(Y)$ that is almost peripherally-multiplicative, then there is a homeomorphism $\psi : \delta B \to \delta A$ and a function α on δB so that $(Tf)(y) = \alpha(y) f(\psi(y))$ for all $f \in A$ and $y \in \delta B$, i.e. T is a weighted composition operator where the weight function is a signum function.

We also show that if T is weakly peripherally-multiplicative, and either $\sigma_\pi(f) \subset \sigma_\pi(Tf)$ for all $f \in A$, or, alternatively, $\sigma_\pi(Tf) \subset \sigma_\pi(f)$ for all $f \in A$, then $(Tf)(y) = f(\psi(y))$ for all $f \in A$ and $y \in \delta B$. In particular, if A and B are uniform algebras and $T : A \to B$ is a weak peripherally-multiplicative operator, that has a limit, say b, at some $a \in A$ with $a^2 = 1$, then $(Tf)(y) = b(y) a(\psi(y)) f(\psi(y))$ for every $f \in A$ and $y \in \delta B$.

Also, we show that if a weak peripherally-multiplicative map preserving peaking functions in the sense $\mathcal{P}B \subset T[\mathbb{T} \cdot \mathcal{P}(A)]$ or $T[\mathcal{P}(A)] \subset \mathbb{T} \cdot \mathcal{P}(B)$ then T is a weighted composition operator with a signum weight function. Finally, for function algebras containing sufficiently many peak functions, including function algebras on metric spaces, it is shown that weak peripherally-multiplicative maps are necessarily composition operators.

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