Doctoral Dissertation Defense
"The Szegö Kernel for Non-Pseudoconvex Domains in $\mathbb{C}^2$."

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There are many operators associated with a domain $\Omega \subset \mathbb{C}^n$ with smooth boundary $\partial\Omega$. There are two closely related projections that are of particular interest. The *Bergman projection* $\mathcal{B}$ is the orthogonal projection of $L^2(\Omega)$ onto the closed subspace $L^2(\Omega) \cap \mathcal{O}(\Omega)$, where $\mathcal{O}(\Omega)$ is the space of all holomorphic functions on Ω. The *Szegö projection* $\mathcal{S}$ is the orthogonal projection of $L^2(\partial\Omega)$ onto the space $H^2(\Omega)$ of boundary values of elements of $\mathcal{O}(\Omega)$. These projection operators have integral representations

$$\mathcal{B}[f](z) = \int_\Omega f(w)B(z,w)dw, \qquad \mathcal{S}[f](z) = \int_{\partial\Omega} f(w)S(z,w)d\sigma(w),$$

where $z \in \Omega$. The distributions B and S are known respectively as the Bergman and Szegö kernels. In an attempt to prove that $\mathcal{B}$ and $\mathcal{S}$ are bounded operators on $L^p, 1 < p < \infty$, many authors have obtained size estimates for the kernels B and S for *pseudoconvex* domains in $\mathbb{C}^n$.

In this thesis, we restrict our attention to the Szegö kernel for a large class of domains of the form $\Omega = \{(z,w) \in \mathbb{C}^2 : Im[w] > b(Re[z])\}$. Such a domain fails to be pseudoconvex precisely when b is not convex on all of $\mathbb{R}$. In an influential paper, Nagel, Rosay, Stein, and Wainger obtain size estimates for both kernels and sharp mapping properties for their respective operators in the convex setting. Consequently, if b is a convex polynomial, the Szegö kernel S is absolutely convergent off the diagonal only. Carracino proves that the Szegö kernel has singularities on *and off* the diagonal for a specific non-smooth, *non-convex* piecewise defined quadratic b. Her results are novel since very little is known for the Szegö kernel for non-pseudoconvex domains Ω. I take b to be an arbitrary even-degree polynomial with positive leading coefficient and identify the set in $\mathbb{C}^2 \times \mathbb{C}^2$ on which the Szegö kernel is absolutely convergent. For a polynomial b, we will see that the Szegö kernel is smooth off the diagonal if and only if b is convex. These results provide an incremental step toward proving the projection S is bounded on $L^p(\partial\Omega), 1 < p < \infty$, for a large class of non-pseudoconvex domains Ω.

Dissertation Committee

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